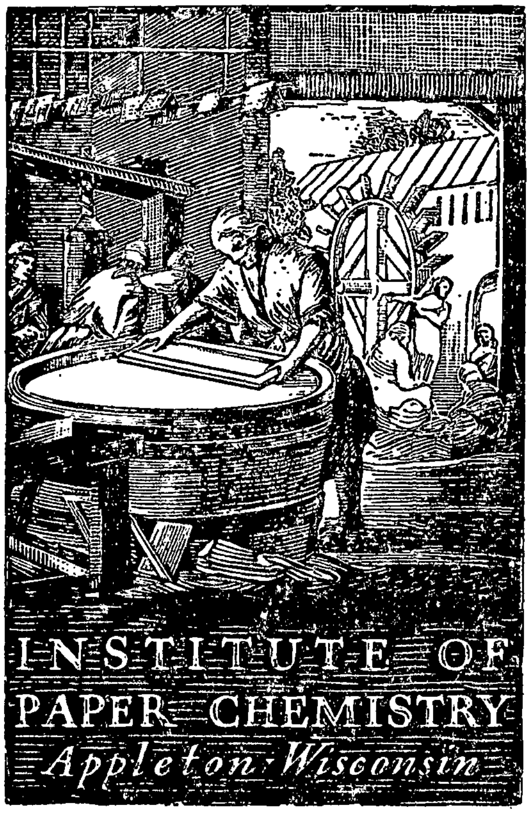

RELATIONSHIP BETWEEN SACK PERFORMANCE AND THE PROPERTIES OF THE SACK PAPER

PART I. THEORETICAL AND EXPERIMENTAL SURVEY OF THE EFFECT OF FATIGUE (Repeated Applications of Stress and/or Strain) ON THE FUNDAMENTAL PROPERTIES OF PAPER

Project 2033

Progress Report Thirteen

to

MULTIWALL SHIPPING SACK
PAPER MANUFACTURERS

March 23, 1960

THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

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SUMMARY

The relationship between the performance of multiwall sacks and the properties of the sack paper has challenged the manufacturers of multiwall sacks and sack papers for many years. In many of the empirical studies of this relationship, fatigue and rupture energy have been the two paper "qualities" which have consistently shown the most promise as being related to sack performance.

As a result of recent experimental studies on sack and sack paper behavior, it is hypothesized that fatigue of sack paper is the manifestation of the deterioration of the fundamental paper properties in repeated tensions. It is further believed that deterioration (and hence fatigue life) of the sack paper is directly relatable to the viscoelastic properties of the paper, that is, the time-dependent, flow aspects of the tensile load-elongation behavior of the paper. For example, based on viscoelastic concepts, it is anticipated that the deterioration of tensile work resulting from repeated stressing of the sack paper, is dependent on

(a) virgin load-elongation properties of the sack paper (including the virgin tensile work)

(b) elasticity properties of the sack paper (of which elastic work is a significant property)

(c) applied work in repeated tension

where the three above-named factors correspond to the <u>same rate of strain</u>. It is believed, therefore, that a study of deterioration of the strength properties of sack paper in repeated tension, based on viscoelastic concepts with due regard to biaxial tension effects and rate of strain, may provide a meaningful approach to the analysis of the performance of multiwall sacks.

As an initial phase of this analysis, an exploratory study was conducted to determine the behavior of sack paper under a progressively increasing number of applications of tensile stress and strain in a laboratory tensile test. Of primary interest are the changes which occur in the tensile properties (load, stretch, work) as a result of repeatedly stressing sack paper under the carefully controlled conditions of a uniaxial tensile test.

Repeated tensile tests were performed in both the in- and cross-machine direction of one sample of 50-lb. kraft sack paper under three controlled conditions in an Instron tester, namely, (a) constant applied load, (b) constant applied elongation, and (c) constant applied work. Three levels of intensity were prescribed for each of these constant processes, corresponding to points on the tensile load-elongation curve where the load was respectively 90, 80, and 70% of the tensile strength of the virgin paper. These levels are referred to as the "high," "intermediate" and "low" levels, respectively, in this report.

For a given level and a given constant process, a specimen was repeatedly loaded a given number of times (short of the number of times required to cause failure) ranging from one to fifteen times. On the final loading of the series the specimen was taken to failure, thereby revealing the ultimate load, stretch, and tensile work available to the sack paper, and thus the change in virgin strength due to the previous applications of load. Also noted for each tensile property (load, elongation and work) were the induced levels associated with repeated stressing according to the given process.

Generally speaking, there was no appreciable change in tensile strength under any of the three constant processes within the range of the number of loadings investigated. On the other hand, the available stretch and tensile work decreased substantially with each of the three constant processes. The change in these properties was greatest as a result of the first application of stress and strain and progressively decreased with successive applications. The higher the level of applied stress or strain, the greater was the decrease in available stretch and work. The deterioration (per cent) of these two properties was more severe for the cross-machine than for the machine direction.

These trends may be explained in terms of the viscoelastic nature of sack paper. The initial stretch may be considered as comprised of an elastic and a plastic component. The plastic component is progressively dissipated as nonrecoverable strain under repeated loading and the residual stretch approaches the elastic component. An analogous explanation holds for tensile work.

It was found that the constant elongation and constant work progresses were virtually equivalent for the sack paper used in this study. That is, repeated application of a constant level of elongation was equivalent to repeated application of a constant work level, and *vice versa*. Thus, the three processes investigated reduce to effectively two: constant force *vs.* constant elongation or work.

Under the constant load process, the sack paper safely sustained a large number of applications of load, even at the high level--in general, beyond the range of this experiment, namely, fifteen applications. Constant elongation or work, on the other hand, frequently caused tensile failure after about five applications. Since the high level, for example, of the several types of processes corresponds to the same point on the initial load-elongation curve, it may be concluded that the constant elongation (or work) process is the more severe type of loading.

Interpretation of the results of this study in the light of current theory suggests that the relationship between sack performance and the fundamental properties of the paper is dependent on the magnitude of the applied stress or strain. For example, when the applied impact stress or strain exceeds the "strength" of the virgin paper, the paper will rupture on the first impact. The mechanism of failure under these conditions does not involve fatigue; thus, failure should be dependent only on the biaxial load-elongation characteristics of the virgin paper obtained at the rate of straining corresponding to that of the applied rate.

In the case where failure results from the repeated application of stress or strain at a level less than the virgin strength, rupture is believed to be associated with the fatigue of the paper in which the latter is a manifestation of the deterioration of the fundamental paper properties. Under these conditions the change in tensile properties as a function of number of loadings may be more indicative of sack paper performance than are the virgin properties obtained from a conventional tensile test. The fatigue phenomenon is believed to occur on a biaxial basis and it has been shown that the cross-machine properties of paper are more sensitive to fatigue than the machine-direction properties of paper. Consequently, the relative importance of the in- and cross-machine properties of the sack paper may change depending on whether the paper is ruptured by a single impact or repeated impacts. Part II of this study is concerned with an investigation to determine whether various samples of sack paper exhibit significantly different rates of deterioration.

INTRODUCTION

The relationship between the performance of multiwall sacks and the properties of the sack paper has challenged the manufacturers of multiwall sacks and sack papers for many years. The numerous studies which have been carried out over the years in an attempt to determine this relationship have met with varying degrees of success. The only general agreement appears to be (a) conventional paper tests such as bursting strength, tensile strength, stretch, etc., do not adequately define the properties of the paper which govern sack performance, and (b) energy considerations, particularly dynamic energy, appear to hold greater promise than the conventional or static tests. As a result of a theoretical investigation of the response of paper to shock loading including rupture energy, Andersson (1) has proposed a new concept for sack paper evaluation, i.e., the impulse tester. In studies carried out by Andersson (1), the impulse test has shown a higher correlation with bag and sack drop performance than conventional tests. In a recent study (2) carried out in this country on pasted and sewn multiwall sacks, it was found that the impulse test did not correlate any better and in some instances not as well as energy measurements obtained by means of "static" tests or a pendulum-type tensile tester. In addition, it was found that the impulse test correlated quite well with rupture energy results obtained by means of the Van der Korput dynamic tensile tester or by integration of the tensile load-elongation curve. This is contrary to the results reported by Steenberg (3) who holds that the impulse test is a unique paper property because it does not correlate with the work of breaking in a pendulum tester.

Ragnossig (4) has considered paper and sack performance on the basis of impact fatigue phenomena and developed the concept of absolute burst energy, i.e., the energy required to just rupture the paper on a single impact. He hypothesized that the absolute burst energy of paper decays with repeated dynamic loading because of progressively increasing severance of the bonds in the paper. From his experimental work he concluded that the decay of dynamic strength (burst energy) was the same for all sack papers and derived the following expression for the relationship between absolute burst energy and the fatigue life as revealed by his impact fatigue test (Frag):

$$BE = \sqrt[3]{N} \, wh$$

where BE = absolute burst energy

N = number of impacts to rupture

wh = applied energy.

In a recent study of the Frag tester (5) it was found that the decay was not constant but varied from paper to paper.

In actual use, a sack conceivably may be subjected to a variety of stresses and strains of varying intensity such that rupture may occur on the first application of "stress" or only after a number of applications, as in a customary sack drop test. In the sack drop test, the sack is subjected to a series of repeated impacts of equal or progressively increasing intensity until the sack fails. Such a sequence of stressing is analogous to a fatigue test and the sack drop test therefore may be considered, at least for the time being, as an instrument for measuring the fatigue characteristics of sacks. It should be emphasized, however, that in considering fatigue of such materials as metals that generally in excess of one-half million to ten million repetitions of loading are necessary to be classed as fatigue (6).

In many of the empirical studies carried out relative to the relationship between sack performance and sack paper properties, fatigue and rupture energy have been the two paper "qualities" which have consistently shown the most promise as being related to sack performance. As a result, it would appear that a logical approach to an understanding of sack performance and its dependency on the fundamental paper properties would be to study the energy absorption characteristics of paper when subjected to repeated application of stress. It is well known that paper exhibits viscoelastic behavior and it may very well be that the deterioration of paper strength under a limited number of repeat loadings may be much more critical than for many other materials.

On the basis of the above, it may be hypothesized that sack performance as measured in a sack drop test is associated with the fatigue of the paper. Fatigue in turn may be merely the manifestation of the deterioration of some of the fundamental paper properties. If we accept the above premise, the problem may be resolved into (a) the determination of the fundamental properties of the papers which are associated with fatigue and (b) the mechanism through which the deterioration of the fundamental properties takes place.

In recent years, a number of studies (7-14) relative to the viscoelastic properties of paper have greatly enhanced the understanding of the fundamental behavior of paper when subjected to stress and strain. It may be helpful to review briefly the dominant features of the stress-strain behavior of paper, in terms of its viscoelastic nature.

The results of a conventional tensile test of sack paper in one direction at a given rate of strain are illustrated in Fig. 1 in the form of a load-

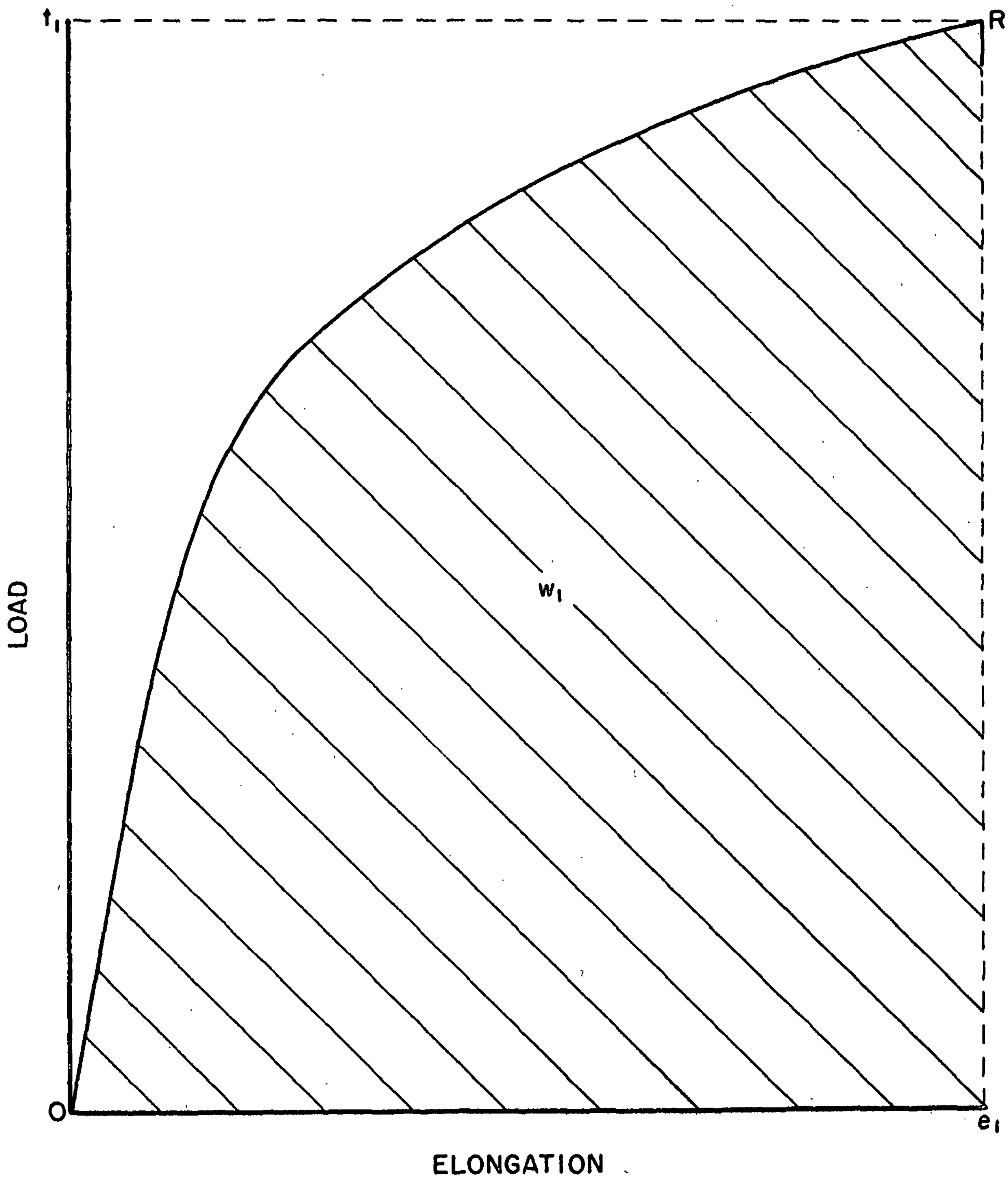

Figure 1. Typical Load-Elongation Curve from Conventional Tensile Test of Sack Paper

elongation curve. The tensile strength at the point of rupture, R, may be denoted by t_1 and the total stretch by e_1. The cross-hatched area under the curve is proportional to the tensile work, w_1, that is, the total strain energy absorbed by the paper up to the instant of rupture. It should be emphasized that the load elongation curve shown in Fig. 1 is the stress and strain response to a given rate of loading. If a different rate of loading had been employed, the response would have been different from that shown in Fig. 1 in accordance with its rheological characteristics (15,16).

Consider now the result of stressing the sack paper to a point short of rupture, such as may occur when a filled sack is impacted (non-destructively) for the first time in a laboratory drop test. The load-elongation behavior (Fig. 2) during this application of stress is represented by the portion $\overline{OA}$ of the original curve. When the stress is removed, the load-elongation behavior will follow the path $\overline{AB}$. Point B indicates that the paper has not returned to its original length but has suffered a permanent deformation of the amount corresponding to $\overline{OB}$. This deformation is frequently called nonrecoverable stretch. Experiments have shown that over a period of time a portion of the nonrecoverable stretch $\overline{OB}$ actually will be recovered, but not in its entirety. Thus, after a period of time the strain state of the material is described by point C.

Suppose the paper, having completed the cycle $\overline{OABC}$, is subsequently stressed a second time. It is found (8) that its load-elongation state will follow a path $\overline{CA}$, whereupon it will resume essentially the same path $\overline{AR}$ which it would have followed the first time if the loading had not been interrupted at A. Usually the curve of the second application of stress will not pass

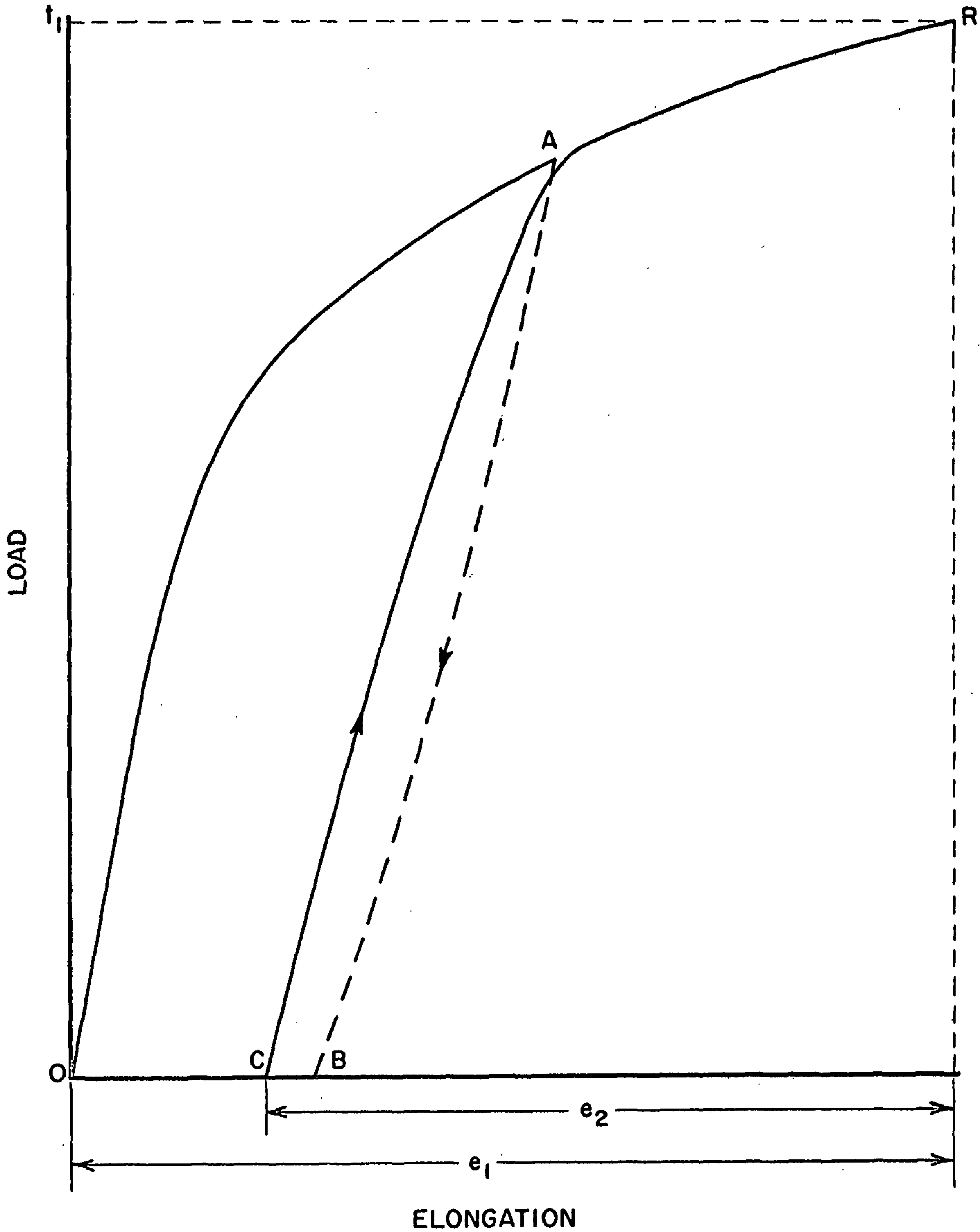

Figure 2. Effect of One Cycle of Stress on Load-Elongation Behavior of Sack Paper

precisely through the point A, but will deviate slightly as illustrated in Fig. 2. Steenberg (8), Hoffmann-Jacobsen (17) and Campbell (18) have reported that the tensile strength exhibited on this second application of stress will not differ markedly from the virgin strength, t_1. On the other hand, the stretch, e_2, available in the paper during the second application is less than the virgin stretch, e_1, by the amount corresponding to the nonrecoverable stretch $\overline{OC}$. Similarly, the available tensile work (the area under the curve $\overline{CAR}$) is less than the virgin tensile work (area $\overline{OAR}$) by an amount equal to the area of $\overline{OAC}$. Thus, it is seen that the capacity of a paper to be stretched or to absorb energy is dependent not only on the rate of stressing but also on the previous stress-history of the paper. It should be emphasized that the sequential removal of available stretch in the sack paper during a drop test or in the field takes place only when the paper is stressed or strained repeatedly below the critical level as defined by the virgin load-deformation curve of the paper. If the magnitude of the stress and strain applied to the paper is greater than the critical level of the virgin paper, the paper will rupture in accordance with the load-elongation characteristics of the paper under corresponding rates of loading. Such a condition is encountered when the first drop in the drop test or in the field is sufficient to cause the paper to rupture on impact.

If the paper is repeatedly stressed, at a progressively higher level, such that several applications may be made prior to rupture, the load-elongation history may appear as in Fig. 3. This illustrates four applications (3 cycles) of stress with rupture occurring on the fourth application. At the start of the fourth application, the stretch available in the paper, $\overline{DF} = e_4$, is considerably less than the virgin stretch, $\overline{OF}$, and the available work, area $\overline{DERF}$, is but a fraction of the virgin tensile work, area $\overline{ORF}$.

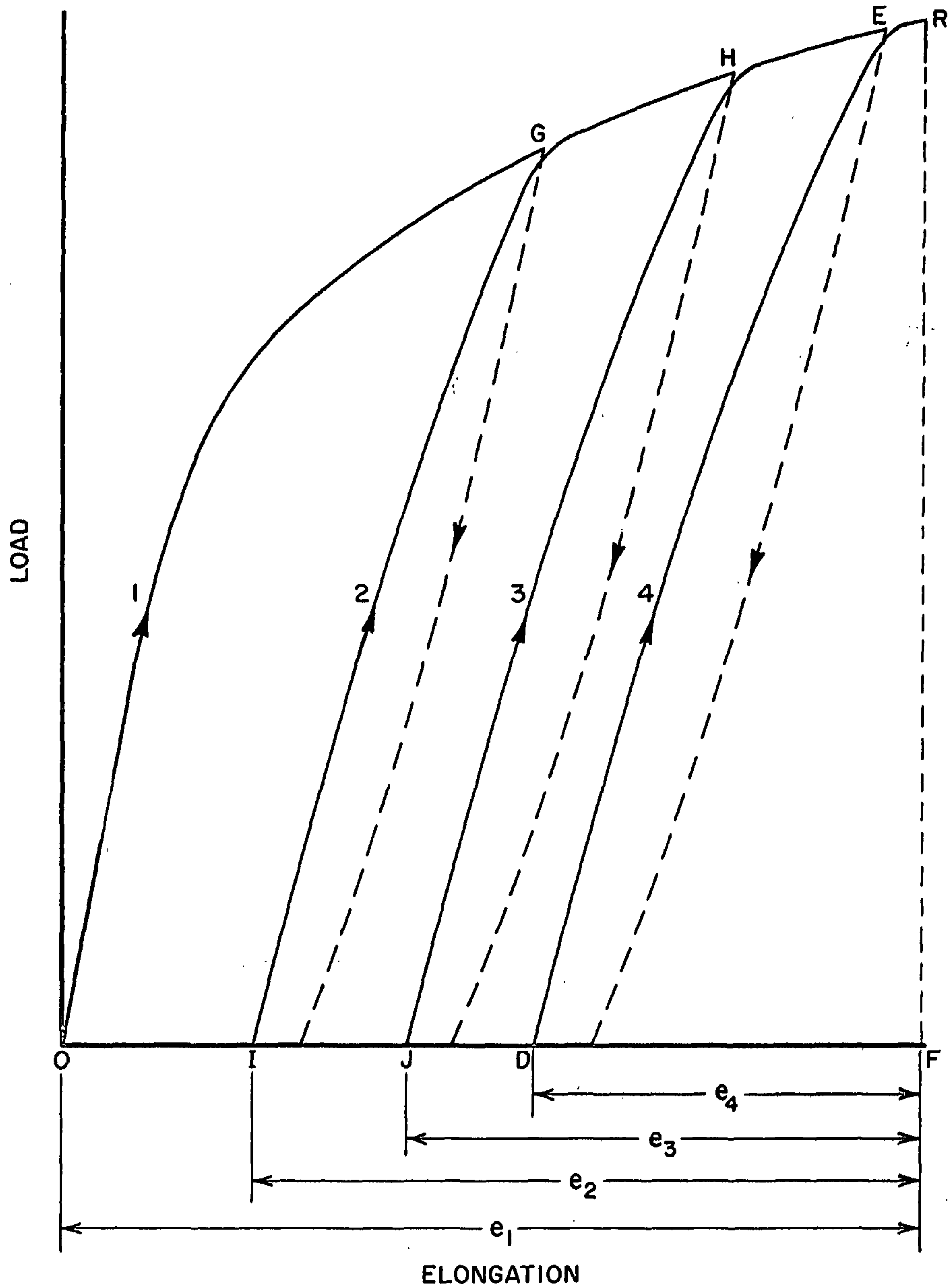

Figure 3. Effect of Multiple Cycles of Stress on Load-Elongation Behavior of Sack Paper

The behavior described above may be viewed as a progressive deterioration of the stretch and work potentials of the paper resulting from each successive application of stress. If one plotted a graph of the stretch which was available in the paper at the beginning of each of the four stress applications of Fig. 3 it would appear as the curve U in Fig. 4. At the beginning of the first application of stress the paper had the capacity to stretch an amount e_1, that is, the stretch determined by a conventional tensile test on a virgin sheet ($\overline{OF}$, Fig. 3). On the second application, however, the available stretch was e_2 ($\overline{IF}$ of Fig. 3), which is less than the virgin stretch by the amount of the nonrecoverable stretch associated with the first cycle ($\overline{OI}$, Fig. 3). Subsequent applications deteriorated the available stretch to progressively lower values as indicated by points on the graph with ordinates e_3 and e_4, i.e., $\overline{JF}$ and $\overline{DF}$, Fig. 3, respectively.

So far no mention has been made of the test routine which was followed in attaining the stress levels G, H, and E in Fig. 3. Figure 3 is drawn for the particular case where the elongations associated with $\overline{OG}$, $\overline{IH}$ and $\overline{JE}$ are all equal, that is, a constant applied elongation. This pattern of applying repeated stress is denoted in the deterioration graph of Fig. 4 by the horizontal dashed line labeled A having ordinate e_a proportional to the constant applied elongation.

It may be noted that just prior to the third application of this level of applied elongation, the stretch available in the paper, e_3, was but slightly greater than the elongation e_a which was to be applied during the third load. Nonetheless, there was sufficient stretch available to safely withstand the third application. The stretch available for the fourth application, namely,

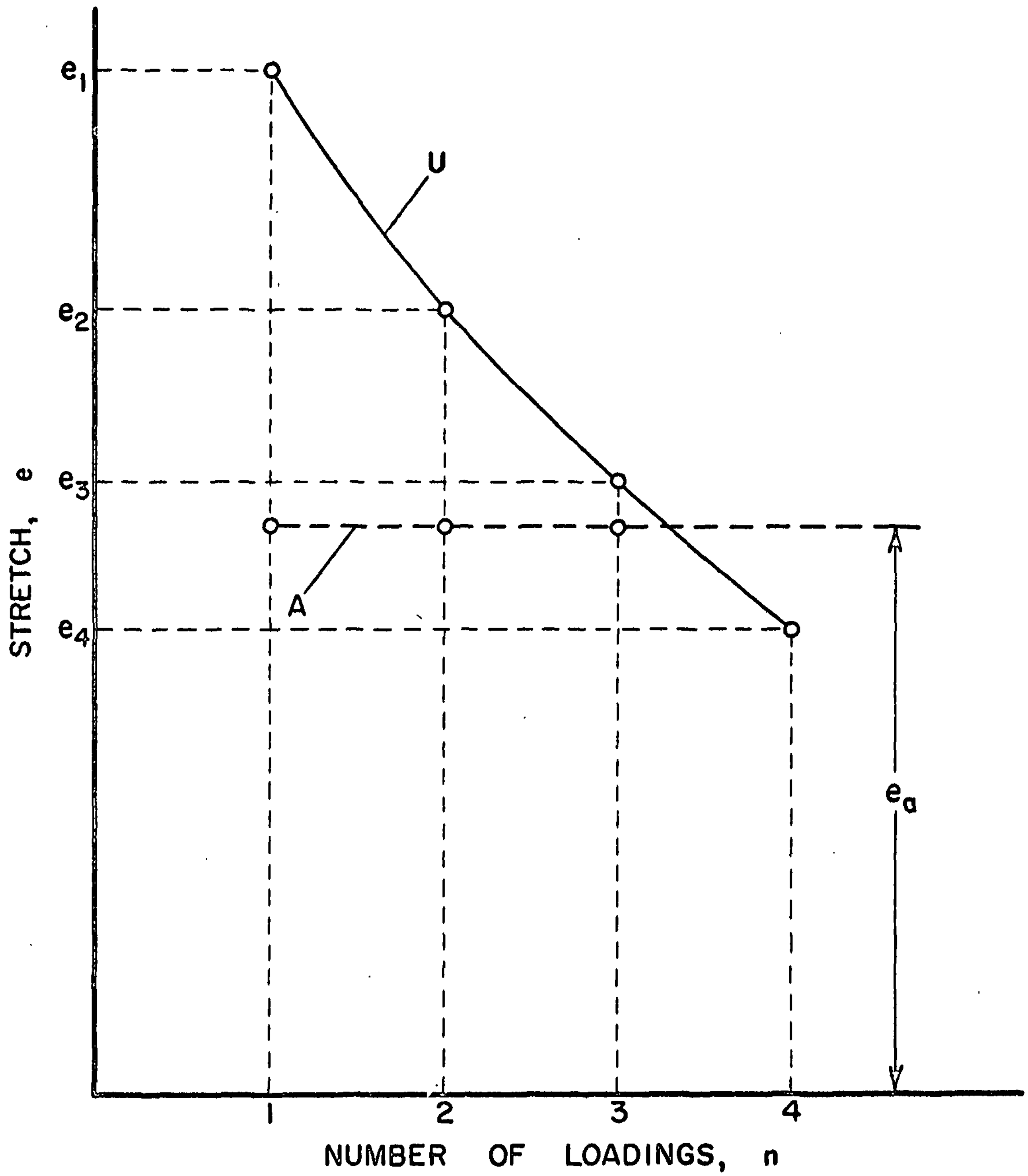

Figure 4. Graph of Deterioration of Paper Stretch as a Function of Number of Applications of Stress

e_4, was less than the intended applied elongation e_a and, consequently, the paper ruptured on the fourth application, as denoted by the intersection of the U and A curves between $n = 3$ and $n = 4$. That is, the stretch of the paper deteriorated to the extent that it was exceeded by the fourth application of applied elongation.

A similar graph may be drawn to represent the deterioration of the tensile work w, resulting from the repeated straining illustrated in Fig. 3. The available work curve is shown in Fig. 5, again labeled U. The work which was induced during each of the first three applications of constant elongation was nearly constant and is represented by the nearly level line A of Fig. 5. Similar to stretch, the ability of the paper to absorb work (energy) deteriorated with each application and eventually was exceeded by the induced work during the fourth application of elongation.

It should be evident that there are an indefinite number of ways of repeatedly stressing a specimen of sack paper. One way would be to progressively increase the tensile load in some definite pattern. Another would be to apply a constant amount of work (area under the curve) in each successive stress application. It may be anticipated that various repeated stress processes will lead to differing types of deterioration curves.

It may be of interest to note that investigations in this laboratory and elsewhere have revealed that a deterioration of tensile properties indeed occurs in the paper of sacks which are subjected to repeated impacts. As reported in Progress Report Four, "Effect of Repeated Impacts on the Strength Characteristics of Sack Paper," (19), the changes in tensile and tear properties

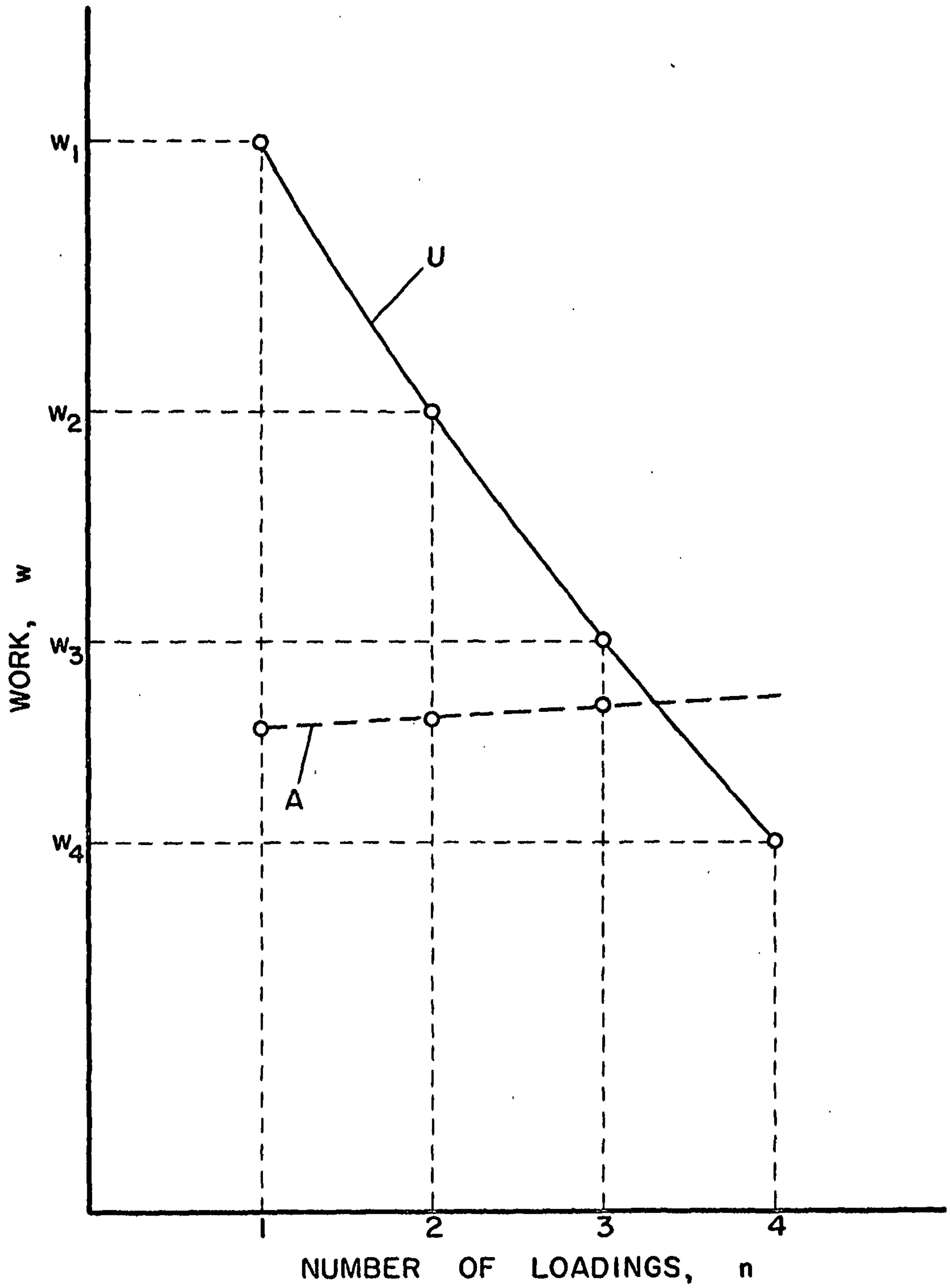

Figure 5. Graph of Deterioration of Tensile Work as a Function of Number of Applications of Stress

of sack paper taken from filled multiwall sacks were measured after the sacks had received a progressively increasing number of impacts. It was found that substantial decreases in tensile properties were incurred, although the tear properties suffered no appreciable deterioration. A recently published paper by Ihrman and Andersson (20) reports a similar type of experimentation on bags with respect to changes in tensile, impulse, and burst properties, wherein substantial decreases in paper strength resulted from repeated impacts of filled sacks.

As mentioned earlier, the concept of deterioration of paper strength underlies the development of the Frag sack paper fatigue tester (4,5). Although the Frag tester records the number of stress applications causing rupture (and hence is a fatigue test), conversion of the test results to "absolute burst energy" is based on consideration of the decay of burst strength under repeated stress application.

In view of the preceding discussion, it seems plausible that the mechanism of fatigue in the case of a viscoelastic material such as sack paper may be a manifestation of the progressive deterioration of tensile work or stretch. It may be worthwhile, therefore, to explore how the concept of strength deterioration may be employed to describe the fatigue behavior of sack paper. For illustrative purposes, attention will be focussed on the tensile work, w, of the sack paper. In view of Fig. 5, it seems reasonable to expect that the fatigue life, n_f (that is, the number of stress applications causing failure) of a sample of sack paper will depend on at least two paper properties and one environmental factor, namely:

(a) virgin tensile work, $\underline{w}_1$

(b) rate of deterioration of tensile work, $\underline{dw}/\underline{dn}$, where $\underline{n}$ is the number of applications of stress

(c) applied work, $\underline{w}_a$.

The intersection of the U and A curves of Fig. 5 describes the fatigue life of the paper. It should be evident that this point of intersection is related to the three quantities cited above. Furthermore, if two samples of sack paper have differing fatigue lives, it should be evident from a study of $\underline{w}_1$, $\underline{dw}/\underline{dn}$, and $\underline{w}_a$. Consider, for example, the hypothetical case of two samples, X and Y, having deterioration properties described by the graph of Fig. 6. These two samples have different values of virgin tensile work, $\underline{w}_1$, but Sample X has the greater rate of deterioration and thus intersects the applied work curve after fewer applications of stress and strain than does Sample Y. That is, the higher virgin "strength" of Sample X was more than offset by its higher rate of decay. In this hypothetical example, Sample Y is clearly the better paper in tensile fatigue.

This example suggests that one way to comparatively evaluate samples of sack paper with respect to fatigue life may be to subject specimens to several cycles of repeated tension and determine the deterioration which has occurred in one or more of its tensile properties as a result of the preceding applications of stress and strain. These data will establish points on the X and Y curves of Fig. 6, which conceivably may then be extrapolated to predict fatigue life without actually subjecting test specimens to a complete fatigue test.

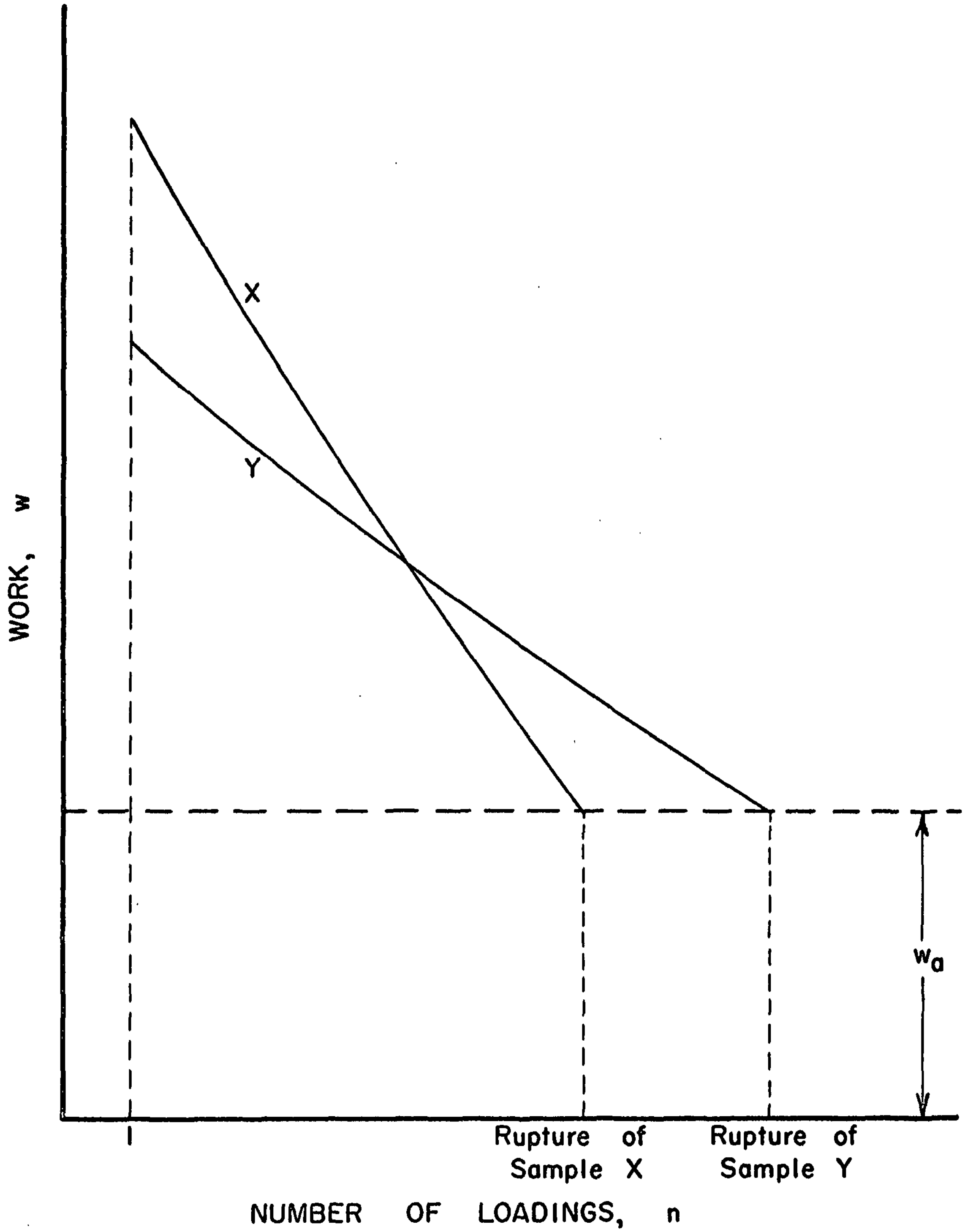

Figure 6. Hypothetical Example of Two Samples of Sack Paper Possessing Different Deterioration Characteristics

On the other hand, it may be of interest to probe deeper into this deterioration concept to see if other possible approaches to fatigue evaluation suggest themselves. Clearly, the study turns on examining the nature of the rate of deterioration. One may ask: On what does the rate of deterioration depend? In addition to its dependence on the number of applications, $\underline{n}$, the deterioration rate may be expected to depend upon (a) the nature of the repeated stress process to which the paper is subjected, and (b) the viscoelastic properties of the paper, for the following reasons. Rate of deterioration is merely another name for the progressive loss in work (or stretch) as a result of each successive cycle of tension. Clearly, the deterioration rate is related to the nonrecoverable work (or stretch) resulting from each application. The nonrecoverable work, on the other hand, is a manifestation of the viscoelastic properties of the paper and of the repeated stress process.

With regard to process, certainly the level of the applied stress, strain, or work may be expected to be a factor. In Fig. 3, for example, it may be readily visualized that an increase in the applied elongation of each cycle would have resulted in a larger nonrecoverable stretch after each cycle and thus a higher rate of deterioration in the stretch and work available in the paper. Thus, the magnitude of the repeated stress process may be expected to be one determining factor in the rate of deterioration of paper strength.

In addition, the type of repeated stress process undoubtedly will affect the deterioration rate. Suppose, for example, that the paper described by Fig. 3 had been repeatedly stressed to a load level corresponding to point $\underline{G}$, rather than by the constant elongation process which led to successive points $\underline{H}$ and $\underline{E}$ on the load-elongation curve. Under constant repeated load, it may be expected that the second and succeeding cycles would be essentially repetitions of the first cycle, as pictured in Fig. 7. Under these conditions, the successive loading curves will be progressively displaced slightly to the right due to creep. The displacement will progressively decrease with each cycle. Presumably, the faster the rate of stressing, the lower the creep. The available stretch and work would be constant after the first cycle if there were no creep and hence the deterioration would be zero after the first cycle. As mentioned above, a small amount of creep does take place as illustrated in Fig. 7; thus, the available stretch and work tend to approach a constant.

Moreover, there are innumerable patterns of repeated applied stress and strain in addition to the constant processes discussed in these examples. Innumerable patterns of incremental increases in load, elongation or work are conceivable. Consider for example sack impact behavior; the constant height drop test and the progressive height drop test undoubtedly are different repeated processes, namely, constant magnitude *vs.* increasing magnitude of stress. Thus, the deterioration rates for a given sample of sack paper may be expected to depend on both the *type* (load, elongation, work) and the *magnitude* of the repeated stress process.

As an aid to exploring the relationship between deterioration rate and the viscoelastic properties of paper, it may be helpful to review further several of the basic concepts of a viscoelastic material. Figure 8 illustrates a typical load-elongation curve, $\overline{OPAR}$, of a classic viscoelastic material strained at a given rate, as discussed in Reference (21). If this material is stressed to point *A* on the virgin load-elongation curve, the total elongation at *A* may be

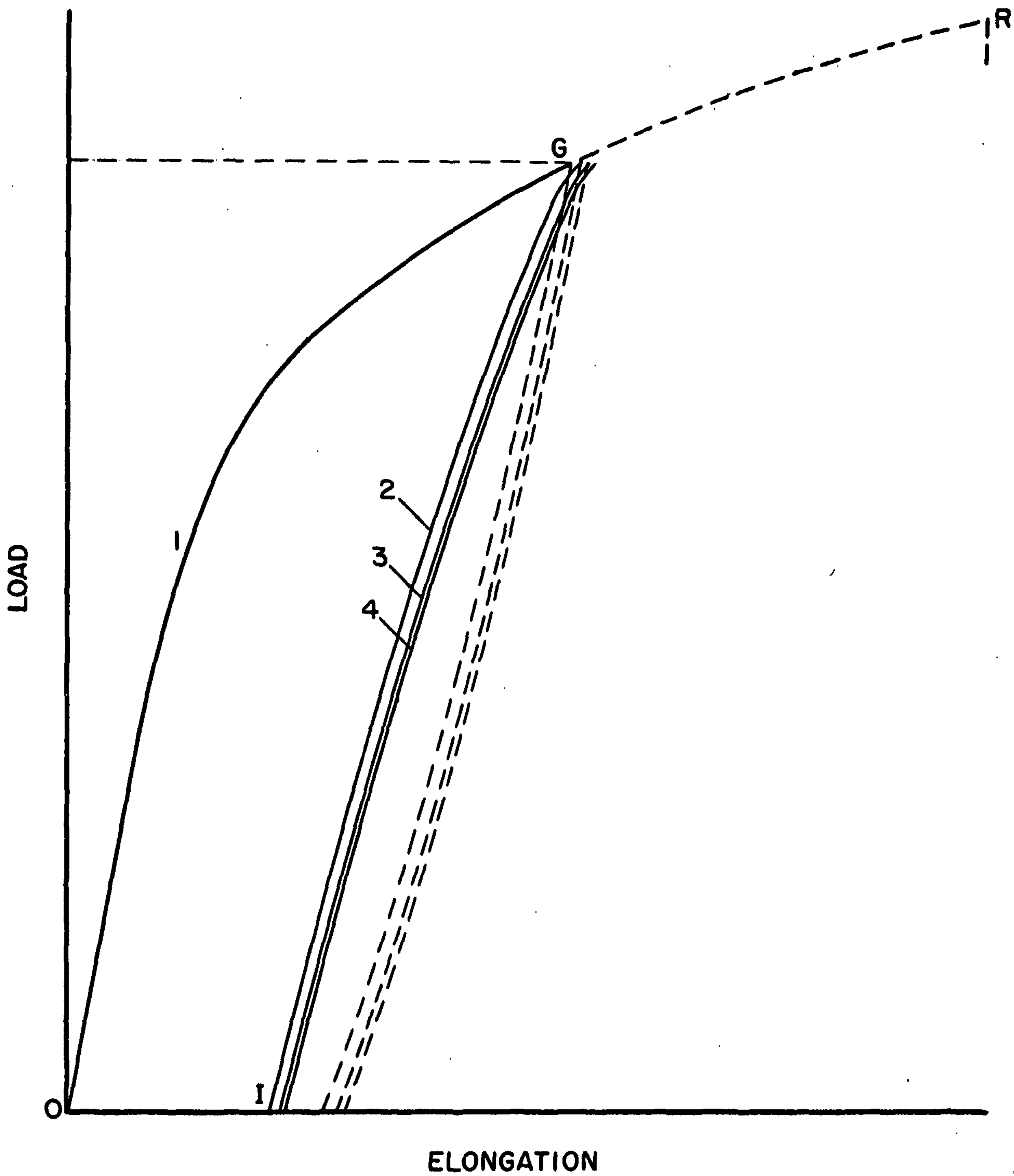

Figure 7. Load-Elongation Behavior Under Repeated Constant Tension Load

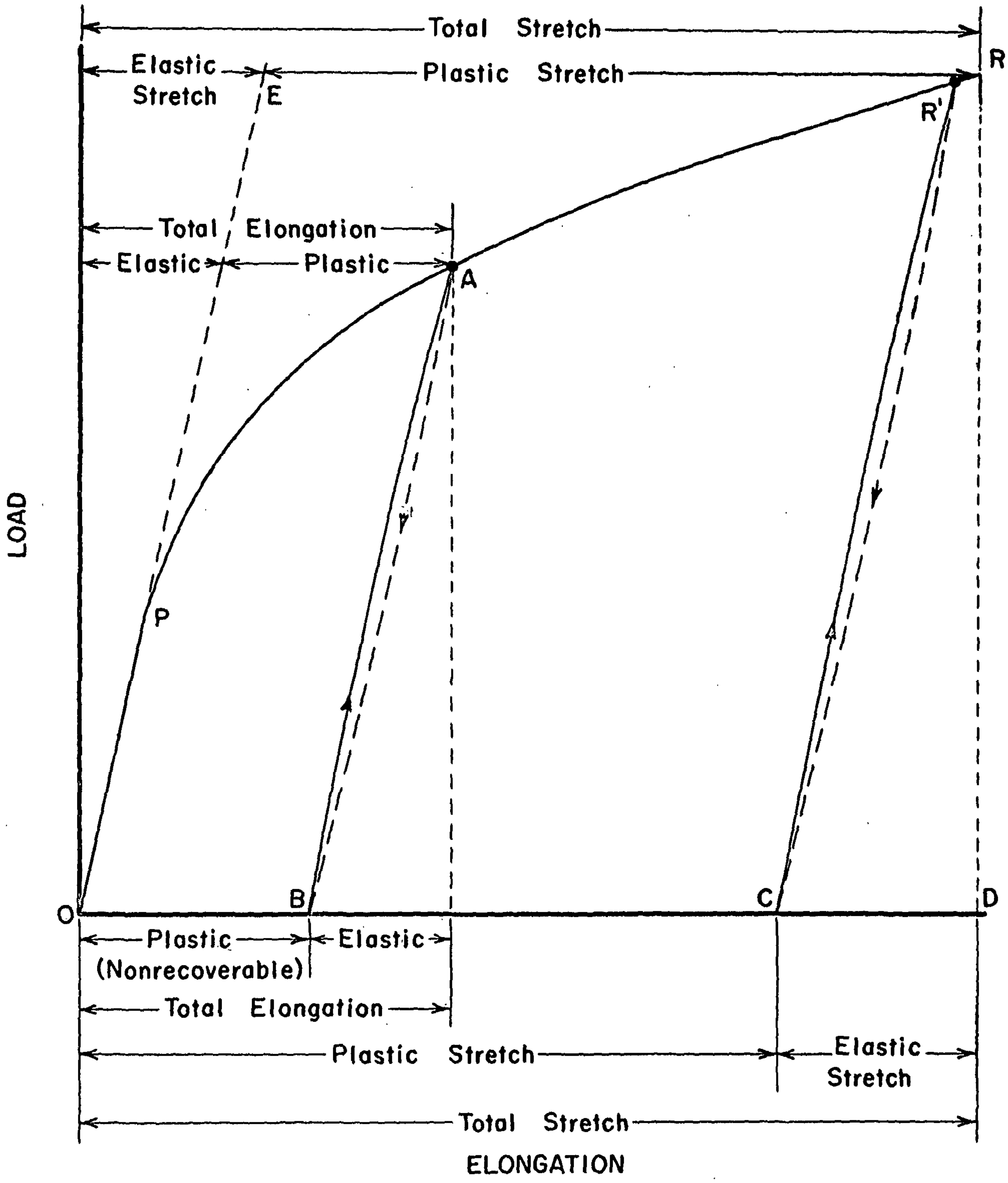

Figure 8. Typical Load-Elongation Behavior of a Viscoelastic Material Possessing Linear Elasticity

regarded as the sum of (a) an elastic elongation and (b) a plastic elongation, as dimensioned above the curve of Fig. 8. The elastic component is the elongation which the material would incur if it could behave in an ideally elastic manner all the way to the stress level of A (the straight dashed line $\overline{OE}$ of Fig. 9) and the plastic component is the additional elongation due to plastic flow. Figure 8 pertains to a linearly elastic material, that is, the elastic component increases linearly with load and total elongation, as denoted by the straight elastic line $\overline{OE}$.

If the load is removed after point A is reached, the load-elongation behavior follows the path $\overline{AB}$ and upon subsequent reloading will follow essentially the path $\overline{BAR}$ (ignoring the slight deviation at A discussed earlier). The nonrecoverable elongation, $\overline{OB}$, after the first cycle is equal to the plastic component of elongation at A, as dimensioned along the horizontal axis of Fig. 8. The elongation induced during reloading from B to A, on the other hand, is equal to the elastic component of elongation induced during the initial loading, $\overline{OA}$. Thus, the plastic component of elongation is lost to the material as a result of cycling, while the elastic component is available to the material for the subsequent loading cycle.

Analogously, the total stretch of the virgin material may be considered to be the sum of an elastic stretch and a plastic stretch, as dimensioned above the curve of Fig. 8. Two aspects of the elastic-plastic behavior represented by Fig. 8 may be noted. First, after the limit of proportionality, P, is exceeded, elastic and plastic elongations occur simultaneously rather than sequentially. (Some materials evidence no proportional limit, in which event, the elastic line $\overline{OE}$ is the tangent to the curve at the origin and a plastic component of elongation is incurred from the inception of loading.) Secondly, the elastic component of *stretch* is not fully utilized until the material has been brought to the point of rupture. Thus, any one loading cycle such as $\overline{OAB}$ involves a portion of the elastic stretch and a portion of the plastic stretch, the latter being dissipated (nonrecovered) at the end of the cycle.

Consider now the behavior of the material when it has been brought to a point, *R'*, arbitrarily near the point of rupture, *R*. This state may be achieved by a single application of load or by a succession of repeat loadings to progressively higher load levels. Unloading will bring the material to the point *C*. Subsequent reloading to point R will rupture the material. The elongation $\overline{CD}$ induced during this final loading is essentially the elastic component of *stretch*, while the nonrecoverable elongation, $\overline{OC}$, prior to the final loading is very nearly the plastic component of stretch. Thus, as *R'* approaches *R*, the entire plastic component of virgin stretch has been dissipated at *C* by the previous loading cycles and the residual stretch in the material is the elastic component of stretch.

In analogy to elongation and stretch, the virgin work, area $\overline{ORD}$, of the material may be regarded as the sum of an elastic component and a plastic component. These components are readily identified in terms of the state of the material at *C* when *R'* approaches *R*. The final loading, $\overline{CR}$, induces an elastic work, area $\overline{CRD}$ (which is equal to the area beneath the elastic line $\overline{OE}$), while the plastic component of work, area $\overline{OARC}$, has been removed during the preceding loading cycle(s).

It seems appropriate to describe the state of the material at *C* as *completely fatigued* for the particular rate of loading employed. At this stage essentially all of the plastic stretch and plastic work have been removed by the preceding applications of stress. A subsequent loading to failure involves only the elastic components of stretch and work. Furthermore, these elastic components are *critical* values of applied elongation and applied work which will just cause rupture of the material under repeated stressing. Greater values of applied elongation and work would cause rupture in fewer cycles, while lesser values theoretically would be incapable of bringing the material to a rupture state.

The preceding discussion has been concerned with the behavior of a viscoelastic material possessing linear elasticity, as denoted by the straight elastic line $\overline{OE}$ of Fig. 8. It has been shown (13) that paper exhibits a somewhat different behavior, namely, nonlinear elasticity, as illustrated in Fig. 9. With paper the elastic curve $\overline{OE}$ is curved rather than straight. Moreover, the elastic *component* of elongation (or stretch) is conceived (13) as the sum of two components: (a) the "immediate" elastic elongation, which is recovered immediately during the unloading phase $\overline{AB'}$, and (b) the "delayed" elastic, which is the time-dependent recovery $\overline{B'B}$. The delayed elastic component is of further significance to creep and relaxation phenomena, which are not considered to be of direct concern to the present discussion.

The nonlinear elasticity illustrated in Fig. 9 for paper poses no obstacle to the previous interpretation of viscoelastic behavior of the classic material illustrated in Fig. 8. The cycle $\overline{OAB}$ removes the plastic component of elongation, $\overline{OB}$, while the nonlinear elastic component is traversed during the reloading from *B* to *A*. The paper is completely fatigued at *C*; all of the plastic

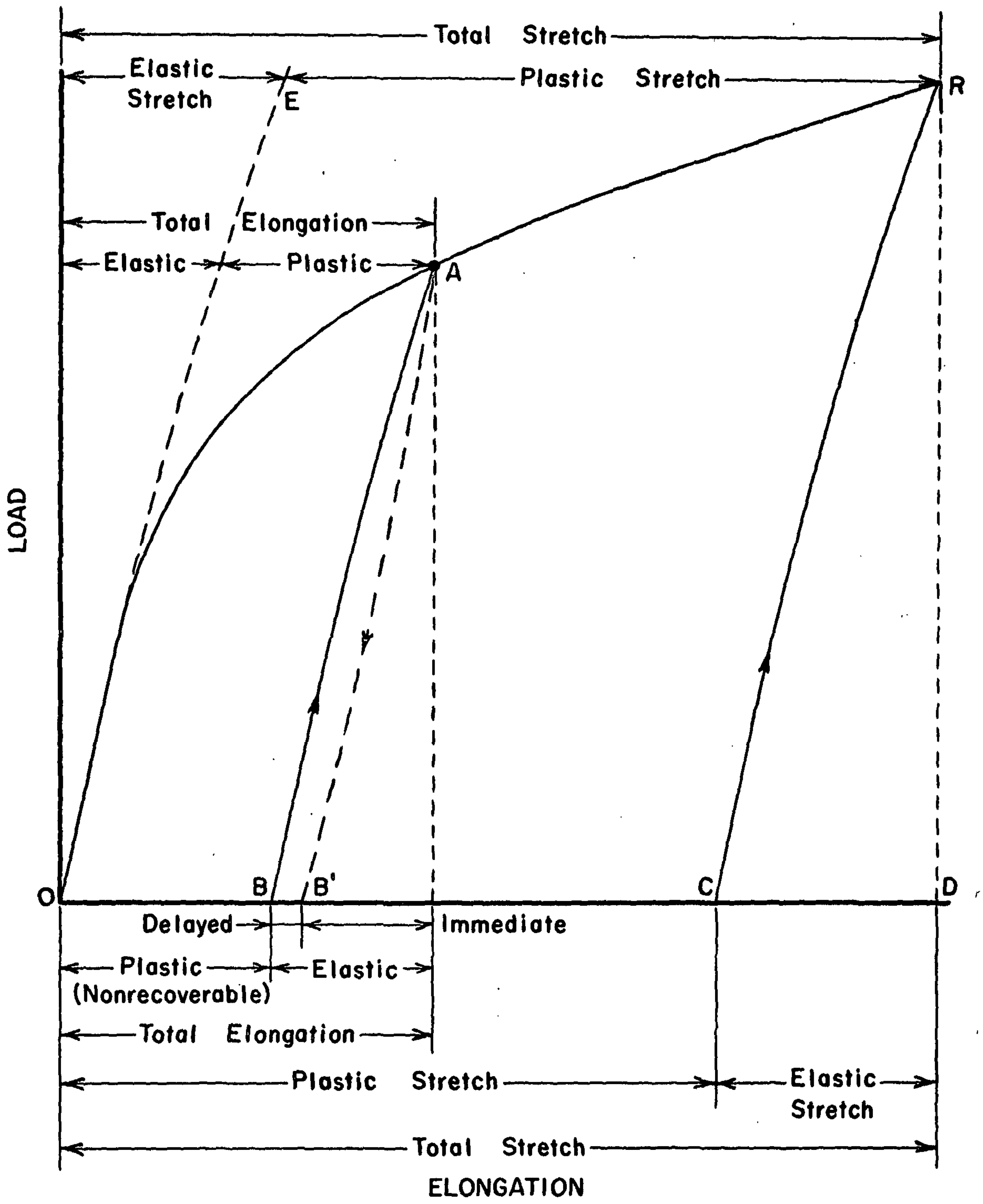

Figure 9. Typical Load-Elongation Behavior of a Nonlinear Viscoelastic Material Such as Paper

stretch and plastic work, area $\overline{OARC}$, have been removed from the paper. An applied elongation $\overline{CD}$, equal to the elastic stretch, will just rupture the material. Alternatively, applied work in the amount of the elastic work, area $\overline{CRD}$,(also the area under the elastic curve $\overline{OE}$) will just rupture the paper.

There is some question whether the reload curves for sack paper, such as $\overline{BA}$ and $\overline{CR}$ of Fig. 9, are strictly parallel. Ihrman and Andersson (20) have reported that reload curves of paper taken from impacted grocery bags showed a progressively lesser slope after successive impacts. Tensile test data obtained in conjunction with the present report indicate that the reload curves after the first cycle are substantially parallel, but have a lower slope than the virgin curve over the same load range. While these results are difficult to explain by current viscoelastic theories, they do not materially alter the interpretation given above. The elastic curve $\overline{OE}$ is really only an artifice for describing elastic and plastic components of elongation; the existence and measurement of these components is made possible only by cycling programs such as those illustrated in Fig. 9. Thus, reload curves such as $\overline{BA}$ and $\overline{CR}$ of Fig. 9 provide the actual basis for definition of elastic and plastic components of stretch and work.

The preceding review of viscoelastic behavior has been presented as a basis for interpreting deterioration phenomena. As a result of cycle $\overline{OAB'B}$ of Fig. 9, the virgin stretch of the paper may be expected to deteriorate by the amount of the plastic elongation associated with point A--i.e., $\overline{OB}$. Similarly, the virgin work may be expected to deteriorate by the amount of the plastic work, area $\overline{OAB}$, associated with point A. When the paper is cycled a second time to a new load level F, higher than A, (see Fig. 10) the paper will acquire an additional increment of elastic elongation, a, and an additional plastic increment, b, as dimensioned above the curve. At the end of the second cycle, G, the increment, b, of plastic stretch will be dissipated and the virgin stretch of the paper will be deteriorated by the amount of the accumulated nonrecoverable elongation, $\overline{OG}$, that is, by the total plastic component ($\overline{OB}$ and $\overline{BG}$) associated with F on the virgin load-elongation curve. Similarly, the virgin work will be deteriorated by the amount of the total plastic component of work associated with F, namely, area $\overline{OAFG}$.

Thus, it may be seen that the deterioration in either stretch (or work) is merely the total nonrecoverable stretch (or work) at the end of a cycle, which in turn is the plastic component of stretch (or work) corresponding to the maximum point attained on the virgin load-elongation curve during the cycle. If the plasticity characteristics of the paper were known, the deterioration behavior under a given repeated stress process could be constructed from the virgin load-elongation curve, obtained at the same rate of strain as involved in the repeated stress process. Inasmuch as the plasticity and elasticity of the paper are complementary, the deterioration characteristics could also be deduced from the (a) elasticity properties (i.e., curve OE of Fig. 10), (b) the virgin load-elongation curve, and (c) the repeated stress process (all factors corresponding to identical rates of strain).

The problem, of course, is how to ascertain the elasticity properties of the paper, because the elasticity curve $\overline{OE}$ is not derivable solely from the virgin load-elongation curve. An approximate solution to the problem possibly may be achieved by experimentally obtaining a "completely fatigued" specimen,

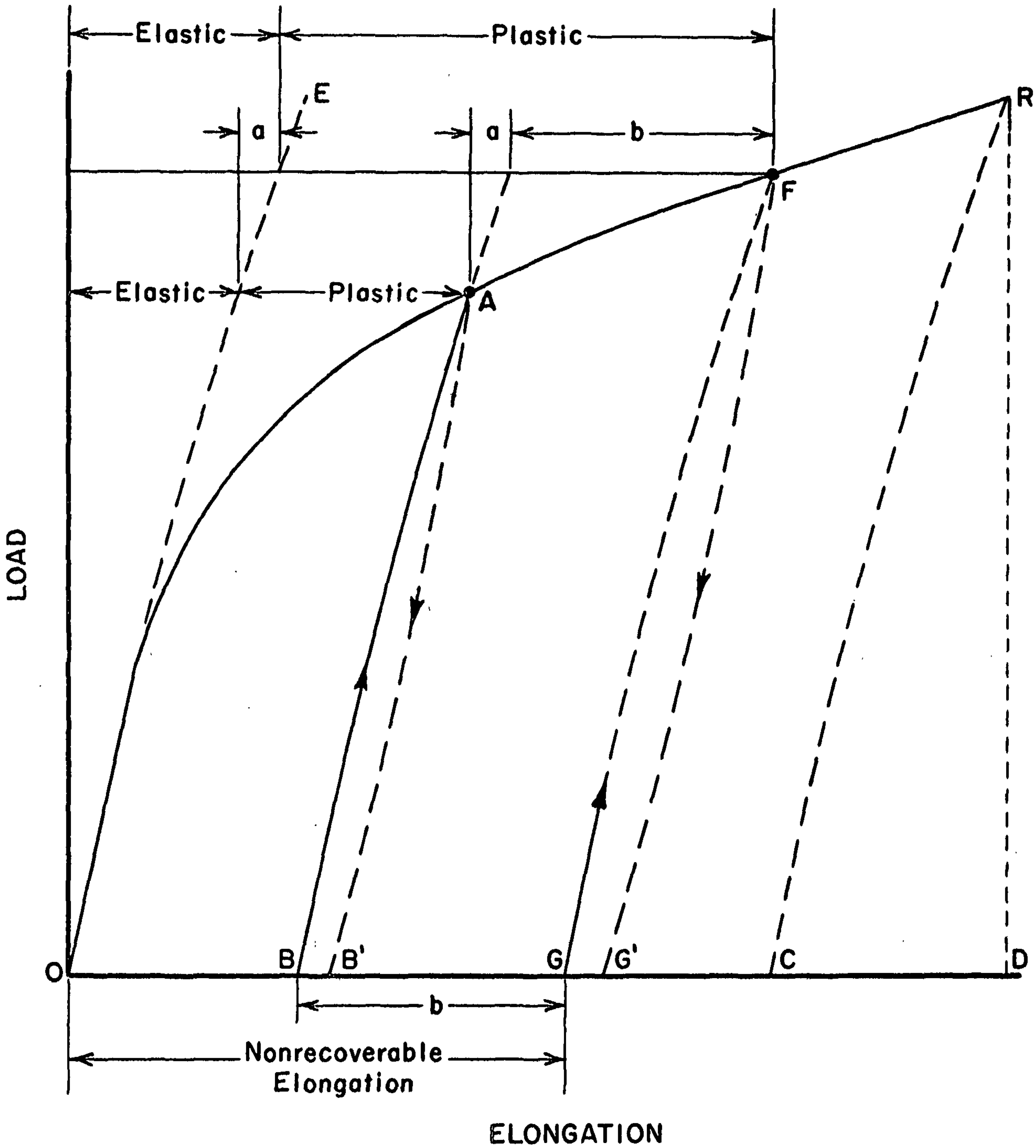

Figure 10. Anticipated Behavior of a Nonlinear Viscoelastic Material Such as Paper in Repeated Tension

corresponding to point C of Fig. 10. For example, after having procured an average virgin load-elongation curve, additional specimens might be stressed to a load level closely approaching rupture, R, unloaded and then reloaded to rupture along a path which is approximately $\overline{CR}$. From this type of experimentation a reasonably good approximation may be obtained for the elasticity properties of the paper, namely: the elasticity curve, $\overline{CR}$; the elastic stretch $\overline{CD}$, and the elastic work, area $\overline{CRD}$.

It should be emphasized that the virgin load-elongation curve of a viscoelastic material such as paper is dependent on the rate of stressing or straining the paper. Steenberg (15) has indicated that when he varied the rate of strain over a range of 1×10^6 the paper became stiffer and behaved more linearly as regards stress response. That is, the tensile strength increased with increase in strain rate whereas the stretch decreased slightly. This phenomenon is illustrated in Fig. 11. Curve $\overline{OR}$ is the load-elongation curve for paper at a given rate of straining. Curve OR' represents the corresponding load-elongation response at a higher rate of straining. It may be noted that at the higher strain rate the tensile strength increased and the stretch decreased. Andersson and Sjöberg (16) in a more recent study have indicated that, over a rate range of 1×10^4, the tensile strength increased with rate of elongation but they could not detect any change in stretch. The authors ascribe the differences in tensile strength to the plastic properties of the sample--the higher the rate of straining, the lower the plastic deformation.

Because of the dependence of load-elongation behavior on rate of strain, it is paramount that the virgin load-elongation curve and the elasticity properties of the paper correspond to the same rate of strain as is involved in the

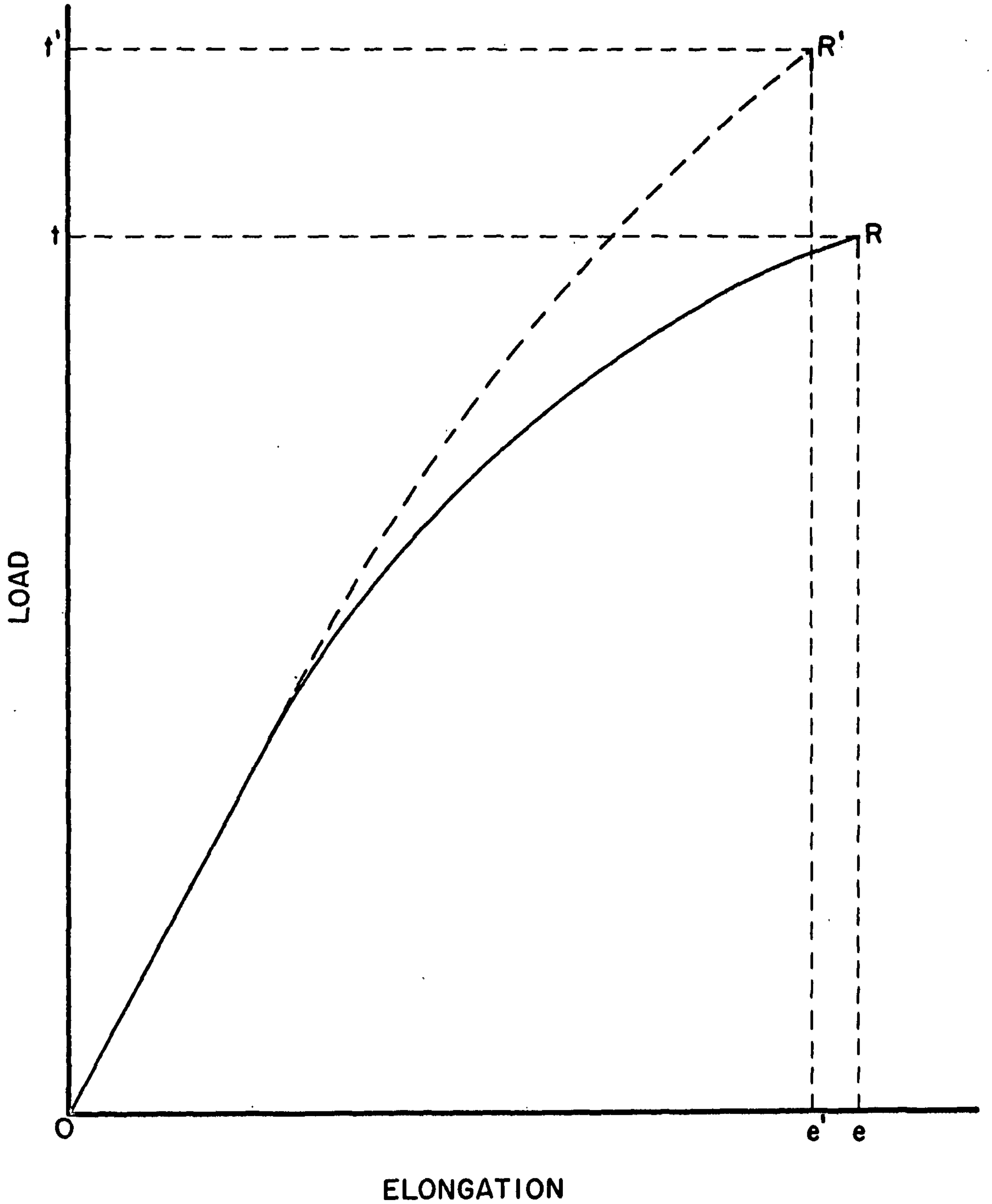

Figure 11. Effect of Rate of Strain on Load-Elongation Properties of Paper

repeated stress process. Only under this condition can it be expected that the deterioration characteristics of the paper are directly relatable to the viscoelastic preparation of the paper.

The foregoing discussion has illustrated the intimate relationship between deterioration properties and viscoelastic behavior of paper. It may be anticipated that the rate of deterioration of sack paper tensile properties is related to (a) the type and magnitude of the repeated tensile process, (b) the virgin load-elongation characteristics of the sack paper, and (c) the elasticity properties of the sack paper, where (b) and (c) correspond to the same rate of strain as (a). Accordingly, the three factors which earlier were presumed to describe the tensile fatigue life of sack paper may now be generalized to the following three factors (phased in terms of tensile work):

(a) virgin load-elongation characteristics (which defines virgin work, w_1)
(b) elastic properties (of which elastic work, w_e, is a significant parameter)
(c) applied work, w_a,

where these three factors all correspond to the same rate of strain. It is believed that knowledge of these three factors would permit construction of a deterioration graph of the type shown in Fig. 5 and thereby describe the behavior of the sack paper under a given repeated stress process.

Analysis of strength deterioration of sack paper due to repeated impacts of a filled multiwall sack probably demands several additional considerations. The preceding considerations have been concerned only with uniaxial tension behavior. On the other hand, it has been shown (22) that the stresses induced in an impacted multiwall sack are of a biaxial (two-directional) nature.

It is believed that the behavior of paper under repeated biaxial tension will probably differ in degree, rather than in principle, from the deterioration concept for uniaxial tension outlined above. It may be anticipated that the behavior of the paper in each principal direction under the action of biaxial tension may be different from its corresponding behavior under uniaxial tension such as is imposed in a conventional tensile test. Relative to uniaxial tension behavior of sack paper, biaxial stresses may be expected to (a) increase the apparent stiffness, (b) decrease the stretch, and (c) change the tensile strength (23). In order to apply a deterioration analysis based on uniaxial properties of paper to sack performance, it will be necessary to have a knowledge of the relationship between uniaxial and biaxial tension behavior of the sack paper.

Furthermore, the strain rate experienced by the paper in an impacted sack is markedly higher than is imposed in most laboratory tensile test instruments. Because of the dependence of viscoelastic behavior on rate of strain, it must be presumed that the virgin load-elongation properties and the elastic properties determined from an Instron test, for example, will not apply directly to the repeated tension process of the sack impact test. This does not preclude, however, the possibility of a favorable correlation between deterioration characteristics of sack paper in a static test and its deterioration in the sack.

Lastly, it should be recognized that the type and magnitude of the repeated stress process of sacks in the field or in laboratory impact tests, e.g., drop test, is not clearly defined at this time. It should be evident that there are innumerable repeated tension processes to which a sack may be subjected in its service life. From the standpoint of laboratory sack performance, one may inquire whether the repeated tension process is best defined or described in terms of load, elongation or work. From the standpoint of the mechanics of an impacted body it would appear that the paper in a sack, repeatedly dropped from a constant height, is subjected to approximately a constant amount of applied energy or work on each impact. Actually the magnitude probably varies with location in the sack as well as slightly from impact to impact. The progressive height drop test on the other hand may be expected to involve progressively increasing applied work, although the increments may not be necessarily constant.

In summary, it is hypothesized that sack performance is associated with the fatigue of the sack paper (except where the sack fails in the first impact), where fatigue is the manifestation of the deterioration of the fundamental paper properties in repeated tension. A review of viscoelastic behavior of paper suggests that deterioration of tensile work, for example, may be directly relatable to

(a) virgin load-elongation curve of the sack paper, which defines virgin tensile work,

(b) elasticity properties of the sack paper, of which elastic work is a significant property,

(c) applied work in repeated tension,

where the three above-named factors correspond to the same rate of strain. It is believed, therefore, that a study of deterioration of the strength of sack paper in repeated tension, based on viscoelastic concepts with due regard to biaxial effects and rate of strain, may provide a meaningful approach to the analysis of the performance of multiwall sacks.

As an initial step in this study, an experimental survey was conducted for the purpose of determining the behavior of sack paper under a progressively increasing number of applications of tensile stress and strain in an Instron tensile tester. The investigation was performed on one sample of 50-lb. sack paper under three controlled repeated tensile processes, namely, (a) constant applied load, (b) constant applied elongation, and (c) constant applied work. The experimentation was arranged to reveal the degree of deterioration in each of three virgin tensile properties (ultimate load, stretch and tensile work) as a function of the number of applications of stress and strain.

The experimental data reported here were obtained during the period December, 1958, to March, 1959, and were reported in part at a meeting of the Technical Committee of the Multiwall Shipping Sack Paper Manufacturers on March 13, 1959 (24). Subsequently, additional experimentation has been accomplished based on the results of this initial study and will be reported as Part II of this series of reports.

MATERIALS

Tensile specimens were prepared from 50-lb. kraft sack paper from Run A-3 of the recent fabrication study (2). This sack paper corresponds to the outer ply of Run A pasted sacks and is the same paper used widely in the instrumentation studies of Project 2033.

TEST PROCEDURE

Twenty-three sheets, 11 inches long and 38 inches wide (roll width), were cut from the parent roll remaining from the fabrication study. The sheets were preconditioned at no higher than 35% relative humidity and 73 ± 3.5°F. for at least 24 hours and then conditioned for at least 48 hours at 50 ± 2% relative humidity and 73 ± 3.5°F.

Each sheet was then further subdivided into an 11 by 22 inch rectangle and an 11 by 16 inch rectangle. Twenty 6-1/2 by 1 inch tensile specimens in the machine direction of the paper were cut from each of the larger rectangles, and twenty cross-machine specimens from each of the small rectangles. Thereafter, the specimens of either orientation were shuffled by sorting into 20 packs and then collected into one pack from which the test specimens were selected.

The specimens were tested at a span of 6 inches on an Instron testing machine equipped with an integrator which recorded the tensile work of the specimen. The crosshead speed was maintained at 0.1 inch/minute for in-machine specimens and 0.2 inch/minute for cross-machine specimens during both the loading and unloading portions of the cycle. The latter test speed was used for convenience because of the high stretch in the cross-machine direction.

A typical test procedure is illustrated by the curves of Fig. 12, which shows the load-elongation record of a specimen subjected to three loadings of a prescribed tensile load. The crossheads were spaced initially so that the specimen was short of sustaining tensile load, corresponding to point **A** of Fig. 12. Then the specimen was loaded from **A** to the prescribed level **B** whereupon it was

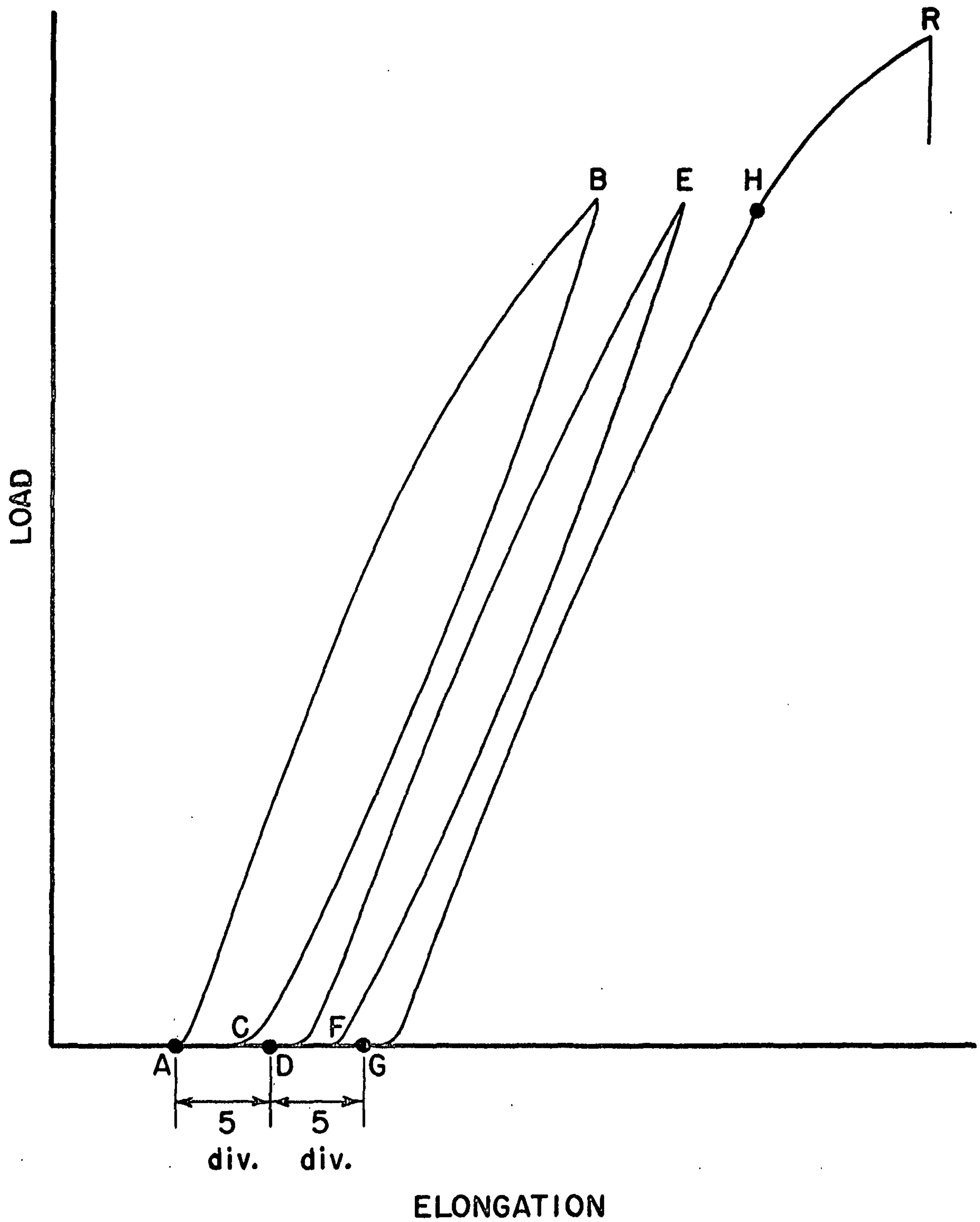

Figure 12. Load-Elongation Curves for Three Applications of a Constant Tensile Force

immediately unloaded at the same deformation rate to point C. The testing machine crossheads were permitted to return to their original positions which corresponds to point A. A recovery period of 15 seconds was employed, during which time the operator manually advanced the chart five small divisions, bringing the pen of the recorder to a point D on the horizontal axis. The chart was advanced so that successive loading and unloading curves could be distinguished easily on the chart from those of the preceding cycle.

At the end of the 15-second recovery period the specimen was loaded a second time to the prescribed load E (which is equal in magnitude to B ~~A~~) and immediately unloaded at the same rate, following the path $\overline{EFD}$, whereupon the crossheads were returned again to their initial position D. The test specimen was permitted to recover for 15 seconds, the chart was advanced to point G, and then the specimen was loaded for the third and, in this case, final time. The final loading was taken to failure, R, passing through the prescribed load level H without hesitation.

An analogous test procedure was employed when the specimen was subjected to n loadings. It may be noted that when a specimen is subjected to n loadings, it sustains (n-1) cycles to a given prefailure level and is loaded to failure on the nth load.

The operator noted and recorded the integrated tensile work corresponding to points B and E of Fig. 12 and the ultimate tensile work corresponding to point R.

The 15-second recovery period between loadings (at points D and G of Fig. 12) was selected arbitrarily and was not intended to simulate the time lapse between drops of a laboratory drop test. Rather, the recovery period was chosen for convenience and standardized at 15 seconds in recognition that paper recovers a portion of its elastic deformation with time. It may be noted that the actual time lapse between the end of one cycle and the pick-up of load of the succeeding cycle was somewhat greater than 15 seconds and progressively so with increase in number of loadings; this occurs because of increasing amount of permanent set which must be passed through before the specimen again picks up load. On the other hand, an identical procedure was followed for each specimen of a sample, so that any effects attributable to the increasing recovery period may be expected to occur systematically with all specimens of a given type subjected to a given number of loadings.

When a specimen was loaded to a prescribed elongation, the test procedure was as described above except that each unloading was commenced when the horizontal distance between the point of rise of the curve from the baseline and the unloading point (B or E of Fig. 12) was equal to the prescribed elongation. When a specimen was loaded to a prescribed work level, the points B and E of Fig. 12 corresponded to a constant value indicated by the integrator.

For the purpose of prescribing constant levels of load, elongation and work, 20 specimens of each orientation were tested by a conventional tensile test. A point on the load-elongation curve was located such that the tensile load, t_a, at this point was 90% of the ultimate tensile strength of that specimen. (See Fig. 13). The elongation, e_a, at this point was determined

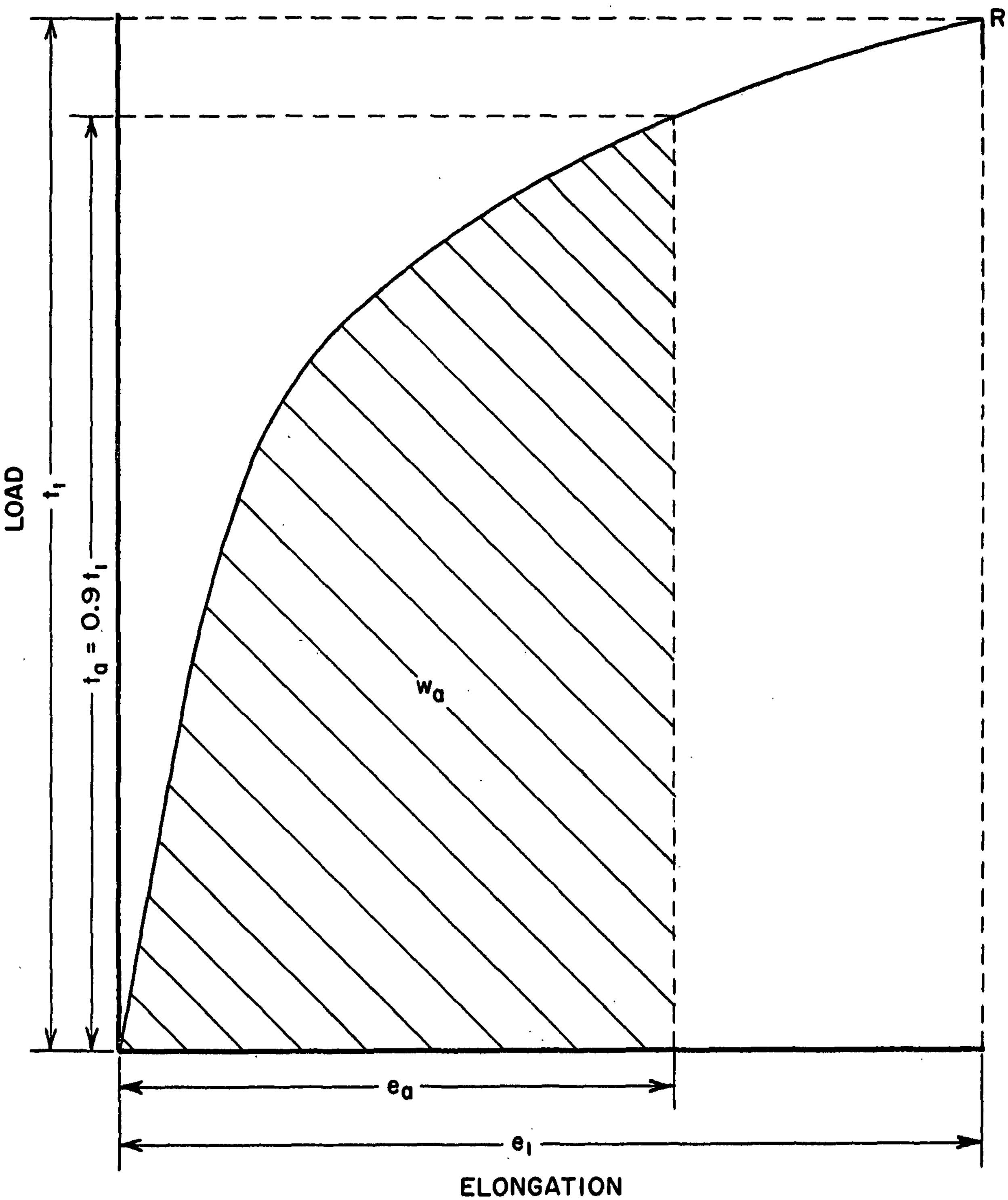

Figure 13. Determination of Applied Levels of Load, Elongation and Work at the 90% Point

and the tensile work, $\underline{w}_a$, corresponding to this point was computed by means of a planimeter. The twenty tensile readings so obtained were averaged, and similarly the elongation and work were averaged. Thus, the constant levels of applied load, applied elongation and applied work corresponding to the "90% point" of the conventional tensile test were established by means of these preliminary tests.

A similar determination was made at the 80 and 70% points of the load-elongation curves. It will be convenient for purposes of this report to refer to these levels as the "high," "intermediate," and "low" levels, respectively. The prescribed levels for each type of loading process are listed in Table I. Also shown in Table I for reference are the ultimate tensile load, stretch, and tensile work of the virgin sack paper.

Having selected these levels for the repeated application of constant load, constant elongation and constant work, a few preliminary repeated tensile tests were performed for the three types of constant processes for the purpose of ascertaining how many cycles the specimen might be expected to survive. Thereafter a test schedule was drawn up detailing the number of applications to which each sample should be subjected. This test schedule is shown in Table II.

As will be discussed in greater detail later, the constant load process at the low level was not performed because the change in tensile properties was too small to be of interest to this study. Furthermore, the cross-machine specimen could withstand no more than one application of the high-level elongation and the high-level work, so that the repeated tensile test could not be performed at these levels.

TABLE I

LEVELS OF APPLIED LOAD, ELONGATION AND WORK

EMPLOYED IN REPEATED TENSILE TESTING

Level	Load, t_a, lb./in.	Elongation, e_a in./in.	Work, w_a in.-lb./in.^{2}
	In-Machine		
High (90% point)	28.0	0.010	0.172
Intermediate (80% point)	24.2	0.008	0.109
Low (70% point)	21.2	0.006	0.072
Ultimate	30.5	0.0125	0.242
	Cross-Machine		
High (90% point)	17.5	0.034	0.457
Intermediate (80% point)	15.2	0.022	0.269
Low (70% point)	13.4	0.013	0.126
Ultimate	19.5	0.045	0.653

TABLE II

TEST SCHEDULE OF REPEATED TENSILE TESTS

Level	Number of Applications		
	Constant Load	Constant Elongation	Constant Work
	In-Machine		
High	1,2,3,4,5,15	1,2,3,4	1,2,3,4
Intermediate	1,2,5,15	1,2,3,5,15	1,2,3,5,15
Low	--	1,2,3,5,15	1,2,3,5,15
	Cross-Machine		
High	1,2,3,5,15	--	--
Intermediate	1,2,3,5,15	1,2,3,4,5	1,2,3,4,5
Low	--	1,2,3,5,15	1,2,3,5,15

The main test program consisted of testing 10 specimens in each direction according to the number of applications of load (or elongation or work) listed in Table II. For example, in the constant load testing of in-machine specimens, ten specimens were loaded the conventional one-load-to-failure; ten specimens were loaded twice (one cycle plus final load to failure); ten specimens were loaded three times (as illustrated in Fig. 12) and so forth. The order of testing was rotated cyclically between these six degrees of loading severity to reduce any systematic error due to possible progressive changes in test operation or test equipment.

It is estimated that the precision of applying a prescribed load during the testing was ± 1%. That is, at the point of load reversal (points B and E of Fig. 12) the actual load was within ± 1% of the prescribed value. The precisions in the constant elongation process and the constant work process are estimated to be ± 2%.

DISCUSSION OF RESULTS

An exploratory experimental investigation was performed for the purpose of evaluating the changes which occur in the tensile properties of sack paper as a function of the number of applications of stress and strain to which it has been subjected in a laboratory tensile test. It is believed that information of this type may lead to a better understanding of the behavior of a filled multiwall sack repeatedly impacted in a laboratory drop test or in actual use.

In- and cross-machine tensile specimens from one run of 50-lb. kraft sack paper from the recent fabrication study were subjected to various intensities of (a) constant tensile load, or (b) constant elongation, or (c) constant tensile work, repetitively applied a progressively increasing number of times.

The load-elongation curves obtained from the repeated tensile tests were analyzed in the following way. If, for example, the specimen was subjected to three applications of load (two cycles plus final load to failure), the ultimate tensile strength, stretch, and tensile work at failure on the third application were read from the curves. This is the point R on the tensile curve of Fig. 12. Tensile work during the last cycle was recorded by the Instron integrator. The sample means of each of these tensile properties was divided by its respective initial value, that is, the tensile strength or stretch or work from a conventional tensile test. Thus, the tensile strength, stretch or work available to the sack paper on the third application of load was expressed as a fractional part of its virgin value.

Furthermore, a point was located on the final curve (the curve of the third load application in the case of Fig. 12) of the loading series corresponding to the prescribed level of the applied parameter. This is the point H on the curve of Fig. 12, which illustrates three applications of constant load. The load, elongation and work at point H were tabulated and expressed as a ratio of the respective virgin ultimate values. Work was obtained by means of a planimeter. The tensile properties at point H have the interpretation of "applied" or "induced" parameters. When, for example, a given load is applied for the third time (point H), a particular elongation and work are induced in the sack paper. Moreover, during the third application of this load, the paper has particular strengths available to it, namely, the tensile properties at point R.

Effect of Repeat Loading on Tensile Properties

The ultimate and applied tensile properties as a function of number of loadings to a constant load level, determined as discussed above, are tabulated in Table III and graphically illustrated in Fig. 14 for the in-machine direction. The virgin properties of this sample, obtained from conventional tensile tests, were:

$$\text{Ultimate Load} = t_1 = 30.5 \text{ lb./in.}$$

$$\text{Stretch} = e_1 = 1.25\%$$

$$\text{Ultimate Work} = w_1 = 0.242 \text{ in.-lb./in.}^2$$

Curves 14a-d pertains to tensile load as a function of number of loadings. The points denoted by "filled circles" (a and c) correspond to the high level of constant load (90% point) as described in Test Procedure. Curve 14a denotes the load ratio at rupture corresponding to the prescribed number of loadings and Curve 14c denotes the values of the applied load ratios.

TABLE III

REPEATED TENSILE PROPERTIES, IN-MACHINE, CONSTANT LOAD

No. of Application	No. of Specimens	Force, lb./in. Applied	Force Ratio[a]	Force Ultimate	Force Ratio[a]	Elongation, in. Applied	Elongation Ratio[b]	Elongation Ultimate	Elongation Ratio[b]	Work, in.-lb. Applied	Work Ratio[c]	Work Ultimate	Work Ratio[c]
						High Level							
1	10	28.0	0.915	31.2	1.019	0.0606	0.807	0.0760	1.012	1.018	0.703	1.505	1.030
2	10	28.0	0.915	31.0	1.014	0.0528	0.703	0.0667	0.888	0.817	0.559	1.230	0.842
3	10	28.0	0.915	31.6	1.032	0.0507	0.675	0.0659	0.877	0.766	0.525	1.235	0.845
4	10	28.0	0.915	31.0	1.011	0.0505	0.672	0.0625	0.832	0.762	0.521	1.126	0.770
5	10	28.0	0.915	32.2	1.053	0.0483	0.643	0.0646	0.860	0.722	0.495	1.236	0.846
15	7	28.0	0.915	31.8	1.038	0.0464	0.618	0.0577	0.768	0.691	0.473	1.034	0.708
						Intermediate Level							
1	10	24.2	0.792	31.2	1.019	0.0462	0.611	0.0776	1.026	0.647	0.439	1.540	1.045
2	10	24.2	0.792	31.2	1.019	0.0437	0.578	0.0752	0.994	0.565	0.384	1.462	0.993
5	10	24.2	0.792	31.2	1.019	0.0423	0.559	0.0699	0.924	0.531	0.360	1.336	0.907
15	10	24.2	0.792	30.4	0.993	0.0428	0.566	0.0684	0.904	0.536	0.364	1.250	0.849
						Low Level							
						Not tested.							

[a] Based on initial tensile strength.
[b] Based on initial stretch.
[c] Based on initial tensile work.

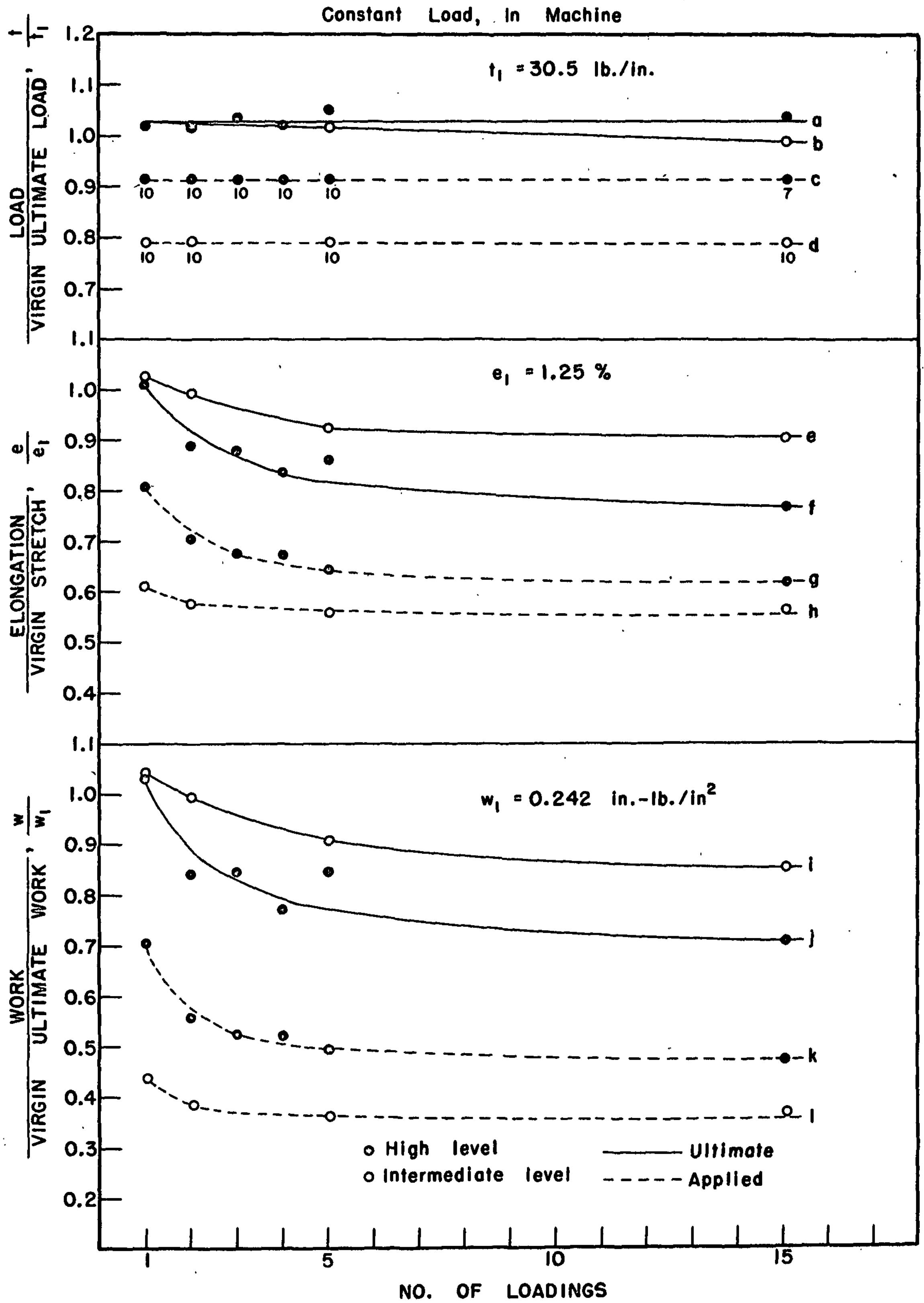

Figure 14. Repeated Tensile Properties of 50-lb. Kraft Paper

(Run A-3), in Machine, Constant Load

Curves 14a and 14c contain the following information: in-machine specimens were repeatedly loaded to a constant load level (horizontal dashed line) equal to about 92% of the initial tensile strength of the sack paper. Ten specimens survived 1, 2, 3, 4, and 5 applications of this load level, while only seven of the ten specimens survived 15 applications of this load intensity as denoted by the numerals beneath the applied curve. It may be noted that the ultimate tensile strength was sensibly constant (see Curve 14a); that is, there was no change in the tensile strength of the paper as a result of repeated applications of constant tensile load.

Inspection of the results for the intermediate level (Curves 14b and 14d) reveals that repeated application of a constant tensile load equal to about 79% of the virgin strength also caused no appreciable change in ultimate tensile strength. The slight downward slope of the ultimate curve for this case is probably not significant.

The low-level application (70% point) was not tested for this process; it would not be expected to show results markedly different from the high and intermediate levels.

The effect of recycling at a constant <u>load</u> level on the elongation characteristics of the paper are illustrated in Curves 14e to h. It may be seen (see Curve 14g) that the first application of the high-level load (approximately 92% of the strength of the virgin sheet) <u>induced</u> an elongation in the paper equal to about 81% of the stretch at rupture of the virgin sheet. The available stretch on the first application is, of course, 100% of the stretch of the virgin sheet (see Curve 14f). The second application of the load induced an elongation in the paper (see Curve 14g) equal to about 70% of the stretch of the virgin sheet. The difference between the first and second induced elongations is attributable to the permanent set (nonrecoverable elongation) suffered by the paper. The stretch available in the paper after the first load cycle and prior to the second application of load was about 89% of the stretch of the virgin sheet (see Curve 14f). Subsequent application of the constant force resulted in continued deterioration of the available stretch in the paper, although at a markedly decreasing rate. The continued decrease in available stretch after the second cycle is believed to result from creep. Had there been no creep, the available stretch after the second cycle would have been expected to be sensibly constant. A similar, though less severe deterioration in stretch at rupture resulted from repeated application of the intermediate level of applied load (see Curve 14e). The induced strain behavior is illustrated in Curve 14h.

The tensile work followed much the same pattern as the elongation under the constant-load processes (see Curves 14i to 1).

Two aspects of tensile behavior are readily apparent from the curves presented in Fig. 14. The downward trend of an ultimate curve (solid line curve) describes the magnitude and rate of deterioration of the "strength" (load, stretch or work) of the paper. Secondly, the vertical distance between the applied curve (dashed line) and the ultimate curve (solid line) represents the margin of strength residing in the paper after it has been stressed a given number of times.

It may be noted that the first point on each of the ultimate curves, corresponding to one application of load, is generally not exactly 1.0, contrary to what one would expect from the definition of the ratio. The reason for this is that the denominator of the ratio (virgin value) was taken as the mean of 30 tests--20 preliminary tests which were performed on the virgin paper to prescribe levels for the three constant processes and 10 tests which were performed on the virgin paper at the time of the cycling tests. The results are tabulated in Table IV. The numerator of the ratio, on the other hand, is the mean of only the latter ten tests. Because of variability, the mean of 10 specimens is not identical with the mean of 30 and hence the ratio is frequently different from 1.0. This apparent discrepancy actually serves a useful purpose. It gives a crude estimate of the variability of the test and the test sample, which may be appropriate to each plotted point.

Because of the variation inherent in tensile testing the observed change in the ultimate tensile strength as a result of cycling below the rupture stress does not appear to be significant. This is in keeping with the results of other investigators (8, 17, 18). The changes noted for stretch and tensile work at rupture are considered to be significant and represent a real change in these properties.

It might be well to mention that the curves of the type shown in Fig. 14 were fitted visually to the plotted points, making use of two criteria. First, an attempt was made to weight the points according to the number of test values represented by the point. Secondly, where a high frequency of ruptures occurred within the range of stress applications studied (1 to 15), the applied and ultimate curves were drawn to intersect at the same number of load applications for load, elongation and work, e.g., Fig. 15. Failure is represented as

TABLE IV

VIRGIN TENSILE PROPERTIES USED IN CALCULATING RATIOS

Process	Level	Tensile Strength, lb./in.	Stretch, in.	Tensile Work, in.-lb.
		In-Machine		
Constant load	High	30.6	0.0751	1.461
	Intermediate	30.6	0.0756	1.473
	Low	--	--	--
Constant elongation	High	30.4	0.0757	1.454
	Intermediate	30.5	0.0749	1.447
	Low	30.9	0.0758	1.492
Constant work	High	30.2	0.0731	1.398
	Intermediate	30.7	0.0751	1.458
	Low	30.2	0.0743	1.426
	Average	30.5	0.0750	1.451
		Cross-Machine		
Constant load	High	19.6	0.272	3.994
	Intermediate	19.4	0.267	3.890
	Low	--	--	--
Constant elongation	High	--	--	--
	Intermediate	19.6	0.270	3.971
	Low	19.4	0.265	3.869
Constant work	High	--	--	--
	Intermediate	19.6	0.265	3.893
	Low	19.4	0.266	3.885
	Average	19.5	0.268	3.917

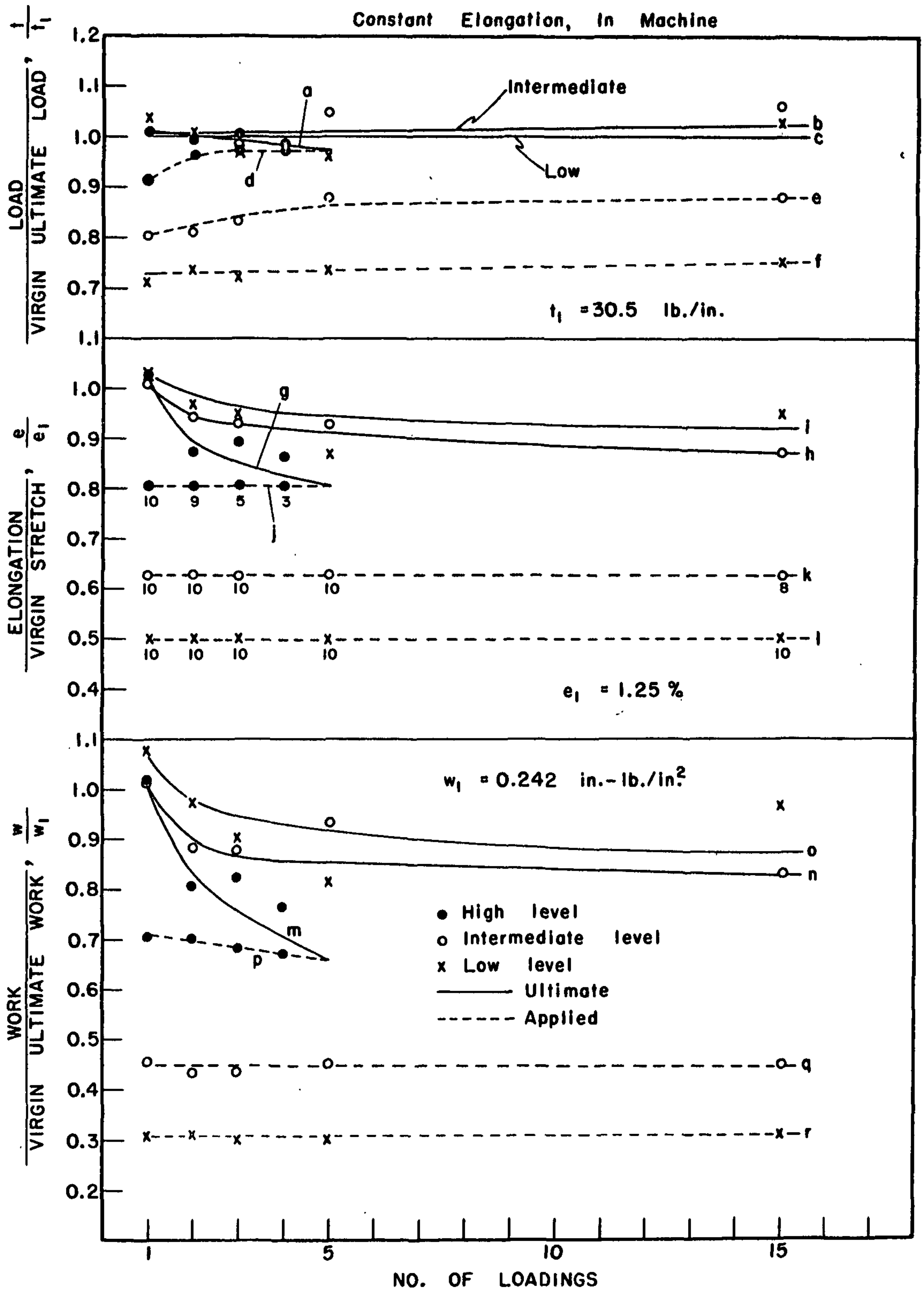

Figure 15. Repeated Tensile Properties of 50-lb. Kraft Paper

(Run A-3), in Machine, Constant Elongation an intersection of the applied and ultimate curves because there is no margin of strength under this condition. Furthermore, when one of the ultimate tensile properties is exceeded (or equalled) by the applied parameter, the remaining two properties are necessarily also exceeded. Thus, the intersection points for the load, elongation and work curves must all occur at the same value of the abscissa.

As previously mentioned, one phase of the present investigation involved the study of the effect of cycling at a constant applied elongation on the tensile properties (tensile, stretch and work) of paper. The procedure followed was the same as was used in the previous section except that a constant elongation was used instead of constant load. Three levels of elongation were used. The three levels of applied elongation used in the machine-direction tests were selected, corresponding to the elongation exhibited by the virgin paper at 70, 80, and 90% of the tensile strength. These levels of elongation were 50, 63, and 81%, respectively, of the stretch of the virgin paper.

The results of cycling at constant applied elongation on the machine-direction tensile characteristics of the paper are tabulated in Table V and graphically illustrated in Fig. 15. The effect of cycling at a constant applied elongation on the tensile strength is illustrated in Curve 15a-f. for the three levels of applied elongation. It may be noted that at the highest level of applied elongation (see Curve 15d), as the number of cycles increase, the induced tensile load increases because of the nonrecoverable elongation. The ultimate tensile strength (see Curve 15a) appears to decrease slightly; however, the decrease is probably not significant. On the basis of the number

TABLE V

REPEATED TENSILE PROPERTIES, IN-MACHINE, CONSTANT ELONGATION

No. of Application	No. of Specimens	Force, lb./in. Applied	Ratio[a]	Ultimate	Ratio[a]	Elongation, in. Applied	Ratio[b]	Ultimate	Ratio[b]	Work, in.-lb. Applied	Ratio[c]	Ultimate	Ratio[c]
						High Level							
1	10	27.7	0.915	30.6	1.007	0.0610	0.806	0.0778	1.028	1.022	0.703	1.485	1.021
2	9	29.4	0.967	30.2	0.993	0.0610	0.806	0.0660	0.872	1.020	0.701	1.175	0.808
3	5	29.6	0.971	30.6	1.007	0.0610	0.806	0.0676	0.893	0.995	0.684	1.197	0.823
4	3	29.3	0.964	30.0	0.986	0.0610	0.806	0.0653	0.863	0.977	0.672	1.115	0.767
						Intermediate Level							
1	10	24.6	0.807	30.8	1.010	0.0470	0.628	0.0753	1.006	0.664	0.459	1.462	1.011
2	10	24.8	0.813	29.5	0.967	0.0470	0.628	0.0705	0.942	0.631	0.436	1.279	0.884
3	10	25.4	0.834	30.1	0.986	0.0470	0.628	0.0696	0.930	0.634	0.438	1.274	0.880
5	10	26.8	0.880	32.2	1.056	0.0470	0.628	0.0696	0.930	0.656	0.453	1.354	0.936
15	8	26.9	0.879	32.4	1.062	0.0470	0.628	0.0652	0.871	0.662	0.457	1.213	0.833
						Low Level							
1	10	22.0	0.713	32.0	1.037	0.0380	0.501	0.0783	1.033	0.463	0.310	1.608	1.078
2	10	22.8	0.739	31.3	1.013	0.0380	0.501	0.0735	0.970	0.464	0.311	1.456	0.976
3	10	22.3	0.722	30.2	0.976	0.0380	0.501	0.0720	0.950	0.450	0.302	1.354	0.908
5	10	22.8	0.739	29.8	0.964	0.0380	0.501	0.0660	0.871	0.455	0.305	1.220	0.818
15	10	23.2	0.752	31.8	1.030	0.0380	0.501	0.0721	0.951	0.462	0.310	1.437	0.963

[a] Based on initial tensile strength.
[b] Based on initial stretch.
[c] Based on initial tensile work.

of specimens which survived progressive application of load or elongation as well as the intersection of the ultimate load curves by the applied or induced load curves, it may be observed that the process involving a constant applied elongation is considerably more severe than the process involving the application of a constant load. It should be emphasized that the applied load and elongation levels of the two processes corresponded to the same point on the load-elongation curve for the virgin paper.

When the low and intermediate levels of applied elongation are considered, it may be seen that the intermediate level of applied elongation resulted in a progressive increase in the induced load (see Curves 15e and f). As would be expected, the intermediate level had a greater effect than the low elongation but smaller than the high elongation level on the magnitude of the change in the induced load, over the range of applications studied. It should be appreciated that rupture would occur at all three levels of elongation (provided they were beyond proportional limit) if sufficient applications were used; however, the number of applications would be inversely related to the magnitude of applied elongation. It may be seen that the ultimate tensile strength level (see Curves 15b and c) was not influenced by the cycling at the low and intermediate levels of applied constant elongation.

The effect of cycling at a constant applied elongation on the stretch characteristics of sack paper is illustrated in Curves 15g-l. It may be observed that in order to obtain a constant applied elongation it was necessary to progressively stress to higher levels. This resulted in a progressive but diminishing nonrecoverable elongation on each cycle. Consequently, the available stretch decreases with the number of cycles (see Curves 15g,h,i). The decrease was most severe for the high level of applied constant elongation and least for the low level.

The effect of cycling at a constant applied elongation on the tensile work characteristics is illustrated in Curves 15m-r. It may be noted that the decrease in tensile work closely parallels the decreases in available stretch. However, the magnitude of the decrease is slightly greater for tensile work. It may be of interest to note that the induced tensile work is sensibly constant for the constant elongation process (see Curves 15p,q,r).

A third type of repeated tension process employed in this study was <u>constant applied work</u>. The test procedure was analogous to those previously described, except that a given level of work was repeatedly applied to the paper specimen rather than a given load or a given elongation. In the machine direction three levels of work were used, namely, 74, 45, and 30% of the tensile work of the virgin paper (corresponding to the points on the virgin load-elongation curve where the load was 90, 80, and 70%, respectively, of the tensile strength).

The results of constant work cycling are tabulated in Table VI for the machine direction of the paper and illustrated graphically in Fig. 16. It may be seen from Curve 16d that cycling to the high level of constant work induced progressively higher load levels in the paper, attributable to the nonrecoverable work of the preceding cycles. This is analogous to the constant elongation process, which also induced progressively higher load levels. Unlike the constant work process, however, there was an apparent increase in the

TABLE VI

REPEATED TENSILE PROPERTIES, IN-MACHINE, CONSTANT WORK

No. of Applications	No. of Specimens	Force, lb./in. Applied	Ratio[a]	Ultimate	Ratio[a]	Elongation, in. Applied	Ratio[b]	Ultimate	Ratio[b]	Work, in.-lb. Applied	Ratio[c]	Ultimate	Ratio[c]
						High Level							
1	10	28.3	0.927	30.1	0.995	0.0604	0.830	0.0701	0.959	1.032	0.738	1.318	0.942
2	9	29.6	0.979	30.4	1.004	0.0617	0.844	0.0673	0.921	1.030	0.737	1.203	0.860
3	9	31.0	1.024	31.8	1.050	0.0608	0.832	0.0652	0.892	1.031	0.737	1.174	0.840
4	4	32.1	1.062	33.1	1.095	0.0595	0.814	0.0640	0.876	1.031	0.737	1.188	0.849
						Intermediate Level							
1	10	24.9	0.811	31.4	1.024	0.0458	0.610	0.0759	1.011	0.652	0.447	1.495	1.025
2	10	26.2	0.855	31.5	1.026	0.0455	0.606	0.0702	0.935	0.652	0.447	1.351	0.927
3	10	26.4	0.858	31.6	1.030	0.0457	0.609	0.0691	0.920	0.651	0.447	1.317	0.903
5	10	26.6	0.868	32.0	1.042	0.0460	0.613	0.0705	0.939	0.653	0.448	1.352	0.927
15	10	27.1	0.884	31.4	1.024	0.0458	0.610	0.0614	0.818	0.651	0.447	1.088	0.746
						Low Level							
1	10	22.2	0.700	30.1	0.995	0.0371	0.499	0.0735	0.989	0.435	0.305	1.402	0.983
2	10	22.4	0.741	30.1	0.995	0.0367	0.494	0.0717	0.965	0.435	0.305	1.397	0.980
3	10	22.8	0.753	32.0	1.059	0.0365	0.491	0.0736	0.991	0.435	0.305	1.474	1.034
5	10	22.3	0.737	30.0	0.981	0.0373	0.502	0.0692	0.931	0.435	0.305	1.284	0.900
15	10	22.8	0.755	30.7	1.015	0.0372	0.501	0.0663	0.892	0.435	0.305	1.253	0.879

[a] Based on initial tensile strength.
[b] Based on initial stretch.
[c] Based on initial tensile work.

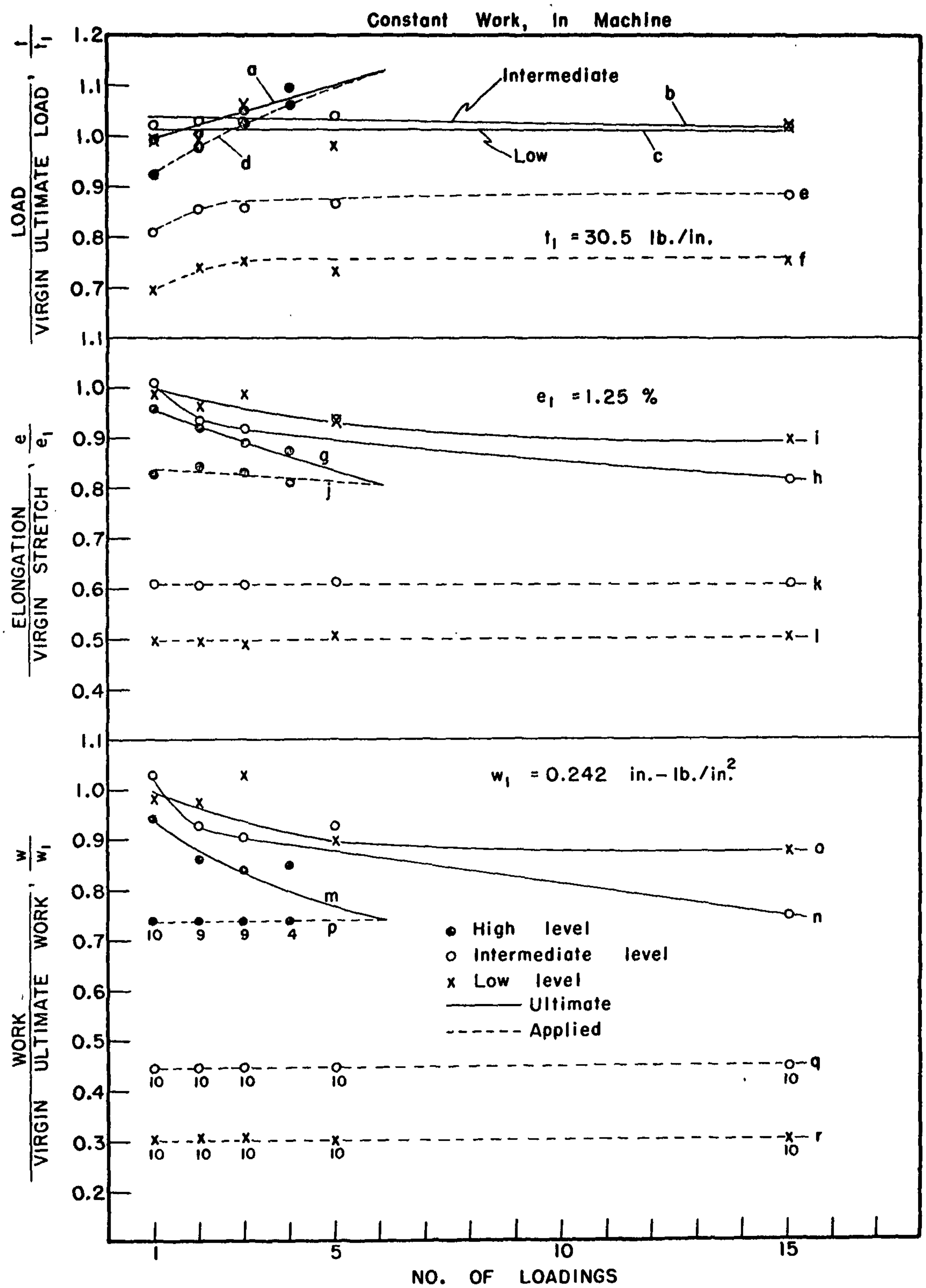

Figure 16. Repeated Tensile Properties of 50-lb. Kraft Paper (Run A-3), in Machine, Constant Work ~~Elongation~~

tensile strength of the specimens as a result of cycling (see Curve 16a). The increase in strength was no greater than 10%, however, and may not be significant. Application of the intermediate and low levels of constant work caused no apparent change in tensile strength but required an increase in the induced load which was most marked during the first several cycles.

From Curve 16g-i it is seen that the stretch progressively diminished with the number of applications of constant work; the greater the level of applied work, the greater the deterioration in stretch. It may be noted that the induced elongation was sensibly constant even though it was the work which was purposely maintained constant during the cycling process. It may be recalled that applying constant elongation (Fig. 15) caused the induced work to be constant. Thus, for this sample of paper, there was an equivalence between the constant work and constant elongation processes.

Curves 16m-o reveal that the tensile work also suffered a progressive deterioration as a result of cycling, with the severity of the deterioration ranked according to the level of the applied work.

Again, it may be noted that the constant work process, like the constant elongation process, was a relatively severe process at the high level. Failure occurred in the majority of specimens on the fourth application. This may be contrasted with the high-level constant-load process which corresponded to the same point on the virgin load-elongation curve but resulted in only a modest number of ruptures in fifteen applications.

Cross-machine specimens from this sample of sack paper were also tested in repeated tension by each of the three processes discussed above. The results of these tests are presented in Tables VII through IX and in Fig. 17 through 19. Inspection of these graphs reveals the same general trends as for the in-machine properties discussed above, although there is a difference in magnitude.

Consideration of all the data reveals that generally speaking there was no appreciable change in tensile strength (load) under any of the three constant processes for either in- or cross-machine tests, within the range of number of loadings investigated. A possible exception occurred under the constant-work process (Curves 16a and 19a). At the higher levels of applied work there is a suggestion of an increase in tensile strength as a result of repeated stressing, although the increase is less than 10%, and may not be significant. By and large, it appears that repeated stressing did not markedly affect the tensile strength of this sample of sack paper, which is in agreement with the results of other investigations (8, 17, 18).

On the other hand, the stretch and tensile work available to the paper always decreased substantially with number of loadings in all the processes employed, and at about the same rate at given levels of a particular process. For example, after 15 applications of the high-level, constant-tensile load, the in-machine stretch was reduced to 77% of the stretch at rupture of the virgin paper and the tensile work to 71% of the virgin paper tensile work. The cross-machine stretch and work were reduced even more severely to 49 and 44% of their virgin paper values, respectively, under fifteen applications of high-level load. Similar trends, though of varying magnitude, may be noted in all of the stretch and work data tabulated in Tables III and V through IX and illustrated in Fig. 14 through 19.

TABLE VII

REPEATED TENSILE PROPERTIES, CROSS-MACHINE, CONSTANT LOAD

No. of Applications	No. of Specimens	Force, lb./in. Applied	Ratio[a]	Ultimate	Ratio[a]	Elongation, in. Applied	Ratio[b]	Ultimate	Ratio[b]	Work, in.-lb. Applied	Ratio[c]	Ultimate	Ratio[c]
High Level													
1	10	17.5	0.894	20.4	1.040	0.1800	0.662	0.287	1.056	2.406	0.602	4.376	1.096
2	10	17.5	0.894	20.4	1.040	0.0990	0.364	0.201	0.740	1.027	0.275	2.978	0.746
3	9	17.5	0.894	20.1	1.028	0.0913	0.336	0.179	0.659	0.912	0.228	2.588	0.648
5	10	17.5	0.894	19.5	0.996	0.0874	0.322	0.153	0.564	0.845	0.212	2.090	0.523
15	9	17.5	0.894	19.8	1.008	0.0796	0.293	0.132	0.486	0.743	0.186	1.768	0.443
Intermediate Level													
1	10	15.2	0.788	19.6	1.013	0.1168	0.437	0.273	1.021	1.236	0.331	4.066	1.045
2	10	15.2	0.788	19.3	1.025	0.0735	0.275	0.217	0.814	0.652	0.168	3.328	0.856
3	10	15.2	0.788	20.3	1.048	0.0698	0.261	0.215	0.805	0.600	0.154	3.486	0.896
5	10	15.2	0.788	19.8	1.022	0.0678	0.254	0.203	0.762	0.559	0.144	3.066	0.788
15	10	15.2	0.788	20.3	1.049	0.0672	0.252	0.222	0.833	0.540	0.139	3.368	0.866

Low Level

Not tested.

[a] Based on initial tensile strength.
[b] Based on initial stretch.
[c] Based on initial tensile work.

TABLE VIII

REPEATED TENSILE PROPERTIES, CROSS-MACHINE, CONSTANT ELONGATION

No. of Applications	No. of Specimens	Force, lb./in. Applied	Force Ratio[a]	Force Ultimate	Force Ratio[a]	Elongation, in. Applied	Elongation Ratio[b]	Elongation Ultimate	Elongation Ratio[b]	Work, in.-lb. Applied	Work Ratio[c]	Work Ultimate	Work Ratio[c]
High Level													
Failure on second application.													
Intermediate Level													
1	10	16.1	0.821	20.5	1.045	0.130	0.482	0.230	1.040	1.555	0.392	4.308	1.085
2	10	17.3	0.878	19.6	0.995	0.130	0.482	0.220	0.816	1.533	0.387	3.210	0.808
3	10	18.3	0.932	19.0	0.966	0.130	0.482	0.160	0.596	1.566	0.394	2.132	0.537
4	9	19.2	0.977	20.3	1.033	0.130	0.482	0.156	0.577	1.632	0.410	2.136	0.538
5	3	20.0	1.018	20.4	1.040	0.130	0.482	0.153	0.568	1.594	0.401	2.076	0.523
Low Level													
1	10	14.2	0.734	19.8	1.022	0.0800	0.302	0.267	1.006	0.788	0.204	4.000	1.034
2	10	15.0	0.772	19.6	1.011	0.0800	0.302	0.235	0.888	0.760	0.196	3.510	0.907
3	10	15.2	0.783	19.6	1.012	0.0800	0.302	0.240	0.907	0.738	0.191	3.582	0.925
5	10	15.5	0.800	18.8	0.966	0.0800	0.302	0.215	0.810	0.713	0.184	2.960	0.765
15	10	17.0	0.875	19.8	1.018	0.0800	0.302	0.167	0.629	0.739	0.191	2.360	0.610

[a] Based on initial tensile strength.
[b] Based on initial stretch.
[c] Based on initial tensile work.

TABLE IX

REPEATED TENSILE PROPERTIES, CROSS-MACHINE, CONSTANT WORK

No. of Applications	No. of Specimens	Force, lb./in. Applied	Force, lb./in. Ratio[a]	Force, lb./in. Ultimate	Force, lb./in. Ratio[a]	Elongation, in. Applied	Elongation, in. Ratio[b]	Elongation, in. Ultimate	Elongation, in. Ratio[b]	Work, in.-lb. Applied	Work, in.-lb. Ratio[c]	Work, in.-lb. Ultimate	Work, in.-lb. Ratio[c]
High Level													
Failure on second application.													
Intermediate Level													
1	10	16.5	0.841	20.3	1.037	0.134	0.507	0.266	1.004	1.614	0.415	4.074	1.046
2	10	17.7	0.904	20.0	1.020	0.133	0.501	0.217	0.819	1.617	0.415	3.224	0.828
3	10	19.1	0.976	20.5	1.050	0.129	0.488	0.177	0.669	1.615	0.415	2.786	0.716
4	5	20.2	1.030	21.2	1.086	0.128	0.482	0.165	0.625	1.617	0.415	2.424	0.623
5	3	20.8	1.061	21.1	1.079	0.127	0.480	0.140	0.529	1.619	0.416	1.920	0.493
Low Level													
1	10	14.0	0.721	19.9	1.025	0.0784	0.293	0.270	1.015	0.758	0.195	4.048	1.042
2	10	14.9	0.767	19.6	1.006	0.0794	0.297	0.236	0.887	0.763	0.196	3.496	0.900
3	10	15.7	0.807	20.5	1.055	0.0800	0.301	0.241	0.906	0.765	0.197	3.612	0.930
5	10	16.3	0.840	20.3	1.045	0.0802	0.301	0.218	0.820	0.759	0.195	3.284	0.845
15	10	17.4	0.892	19.8	1.020	0.0808	0.305	0.168	0.632	0.762	0.196	2.396	0.617

[a] Based on initial tensile strength.
[b] Based on initial stretch.
[c] Based on initial tensile work.

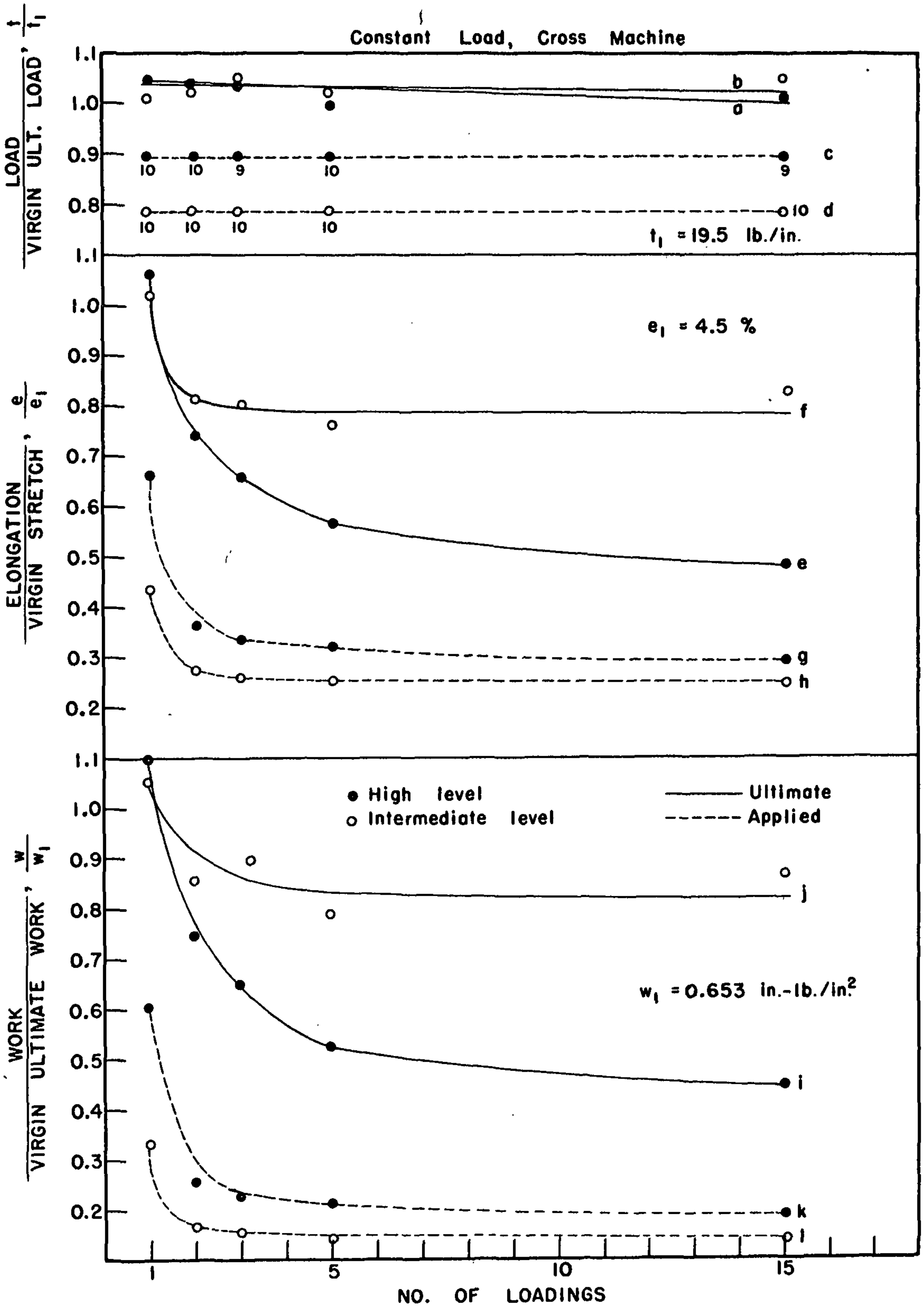

Figure 17. Repeated Tensile Properties of 50-lb. Kraft Paper

(Run A-3), Cross Machine, Constant Load

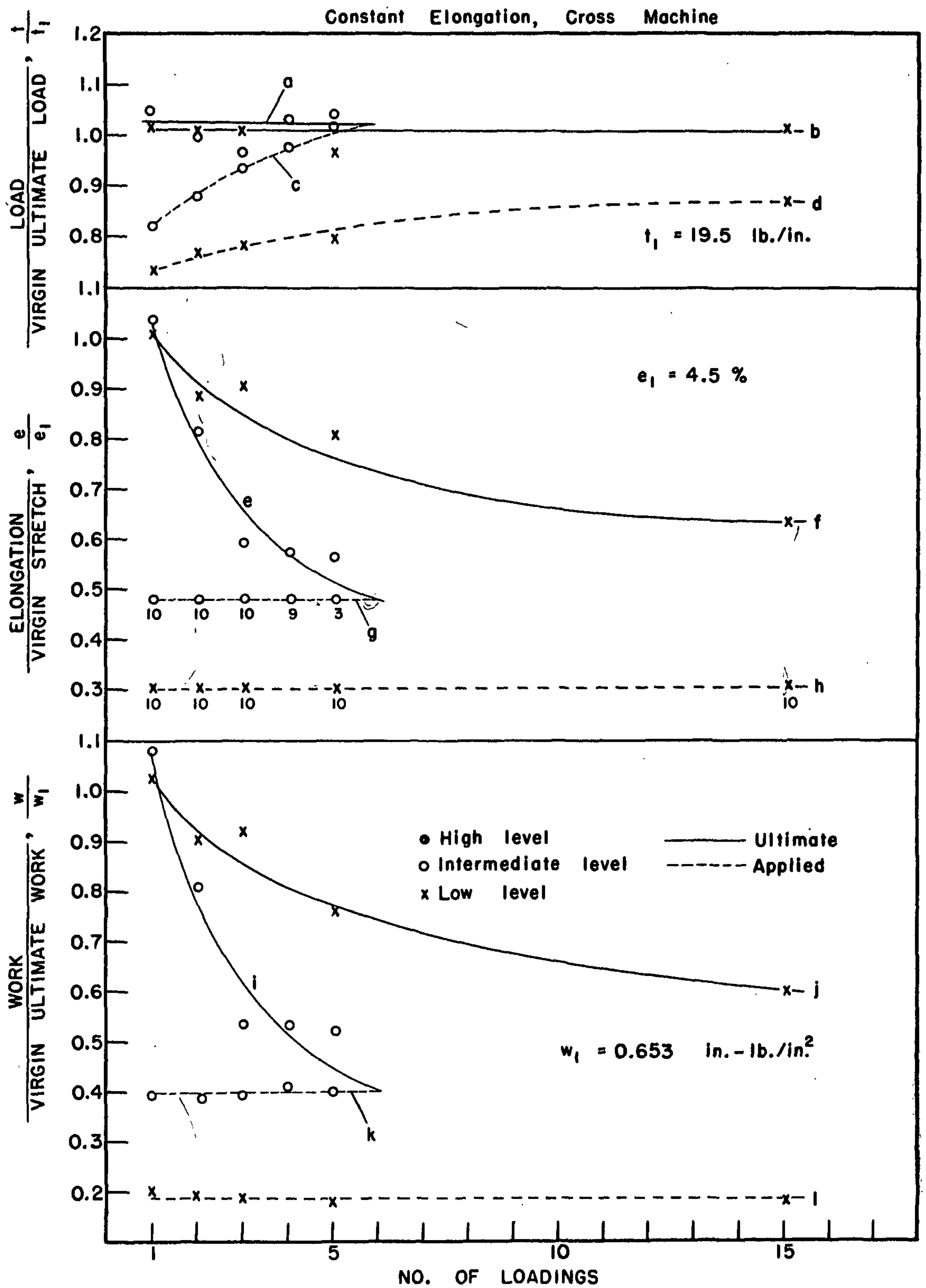

Figure 18. Repeated Tensile Properties of 50-lb. Kraft Paper (Run A-3)

Cross Machine, Constant Elongation

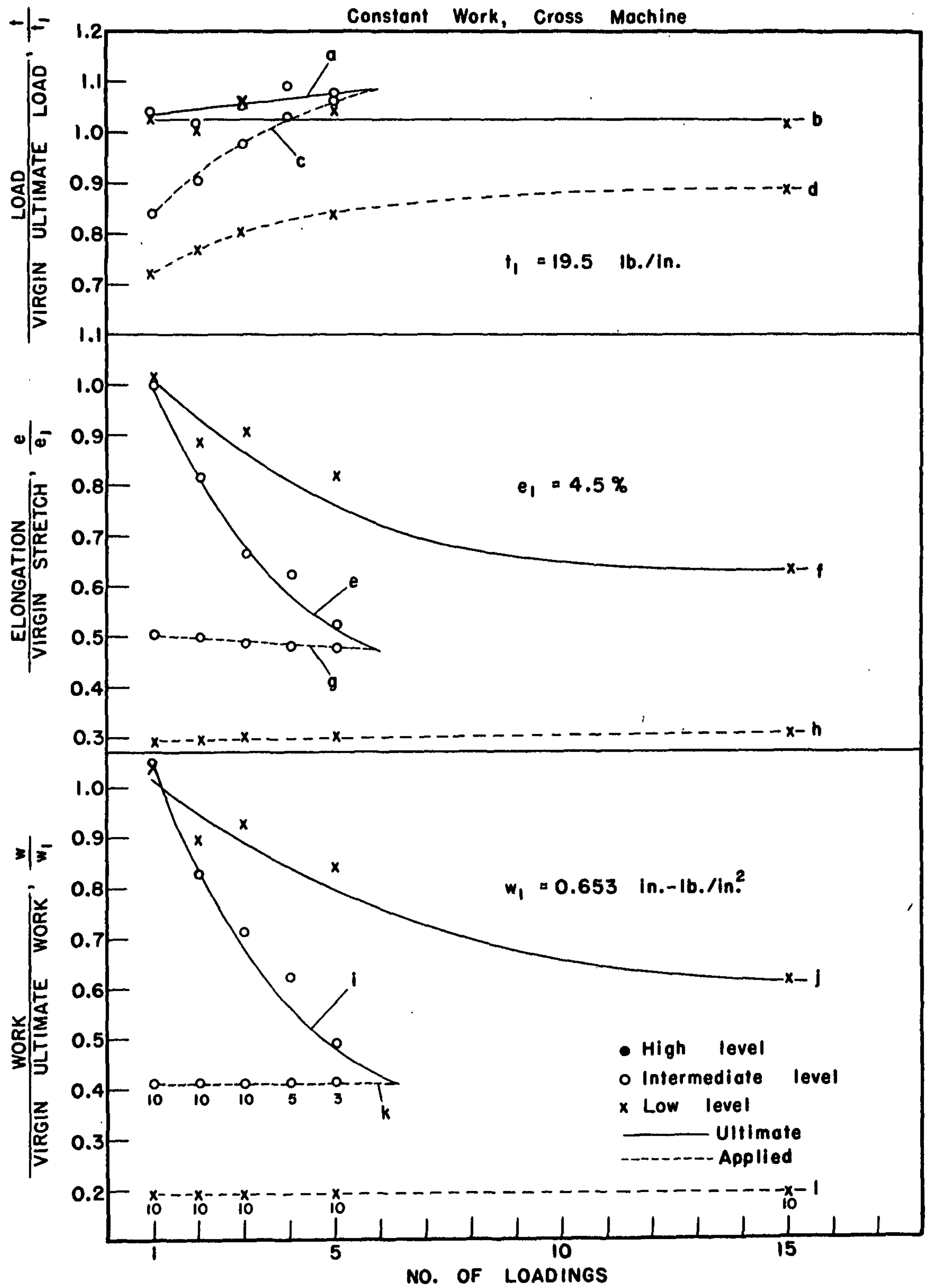

Figure 19. Repeated Tensile Properties of 50-lb. Kraft Paper (Run A-3),

Cross Machine, Constant Work

The decrease in stretch may be anticipated from considerations of nonrecoverable stretch as discussed in the introduction to this report. As illustrated in Fig. 20, the result of the first application of load (cycle $\overline{OAB}$) is a permanent set $\overline{OB}$ (nonrecoverable stretch). The stretch, e_2, available to the paper on a second application of load is less than the virgin stretch, e_1, by the amount $\overline{OB}$. Tensile work is proportional to the area under the loading curves of Fig. 20. The work available to the paper during the second application of load is the area $\overline{BARC}$ and is less than the virgin work by the area $\overline{OAB}$. Since the branch $\overline{AR}$ of the virgin load-elongation curve is not altered appreciably by the cycling process, it is reasonable to expect that the available tensile work will roughly parallel the available stretch as a function of number of stress applications.

It may be noted that in all processes the rate of decrease in available stretch and work was greatest on the first application of load, elongation or work and thereafter progressively decreased. After the first several applications of stress, the ultimate curves frequently leveled out and approached the horizontal. Exceptions are the high level of input wherein rupture occurred within the range of the number of loadings investigated. The rate of decrease in available stretch and work may be explained in terms of the viscoelastic properties of paper as discussed in the Introduction. The virgin paper stretch, e_1, may be considered as made up of an elastic stretch component and a plastic stretch component. The plastic stretch is progressively lost in the form of nonrecoverable strain upon repeated applications of load. On successive applications, the available stretch approaches ever more closely the elastic stretch. Thus, the rate of decrease in stretch and work is great during the first few applications of stress and then

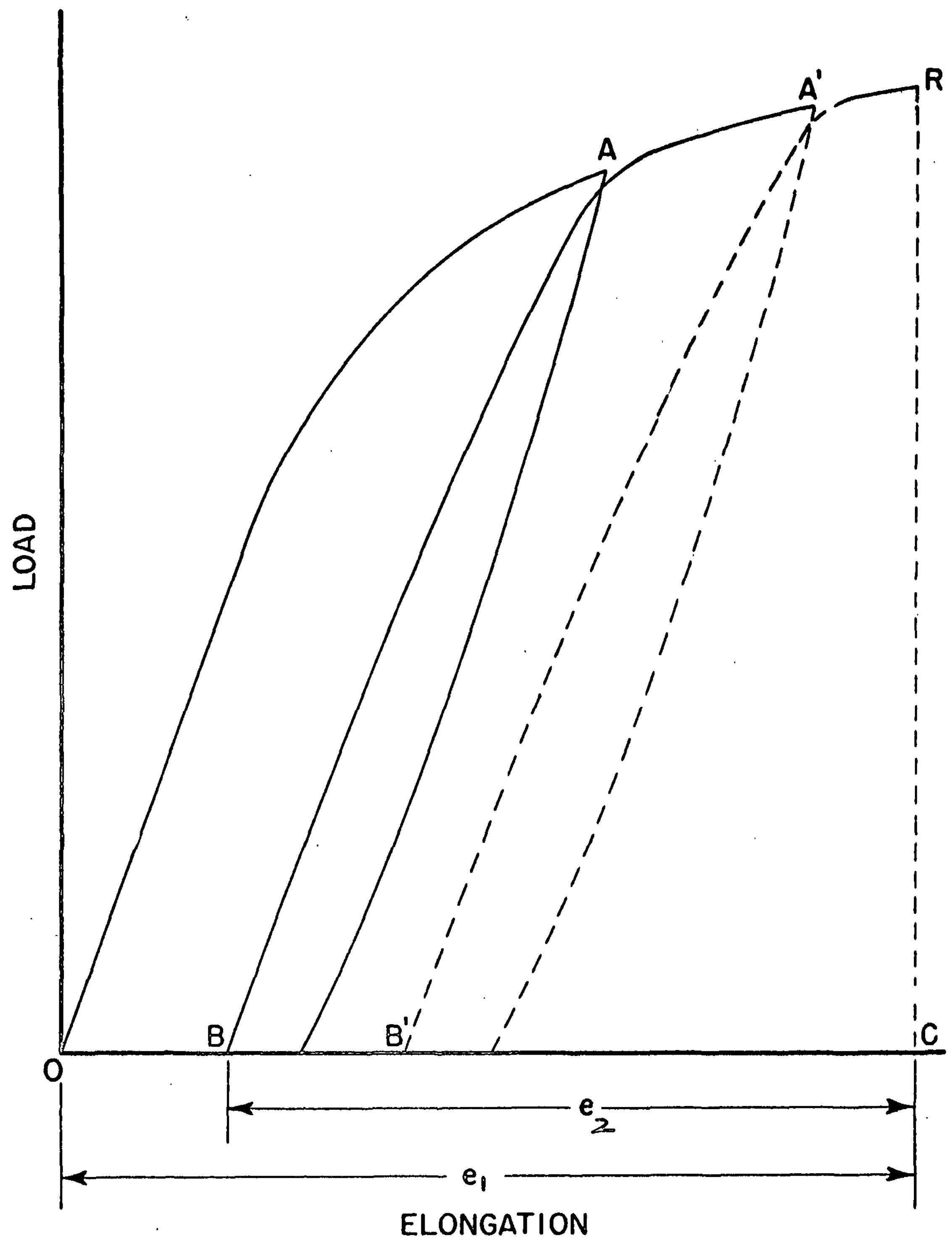

Figure 20. Illustration of Nonrecoverable Stretch in Repeated Tensile Test

diminishes as the plastic stretch becomes dissipated. Of course, if the applied elongation or applied work is relatively large compared to the elastic component, it soon exceeds even the residual elastic stretch and rupture occurs after relatively few cycles as shown in Fig. 15, 16, 18, and 19. Note that there is a marked distinction between the constant-load process, on the one hand, and the constant elongation and work processes, on the other hand, with regard to the incidence of rupture. As discussed earlier, constant applied load seldom resulted in rupture in this range of loadings, even at the highest level of input, while the high and intermediate levels of elongation and work frequently caused rupture within the number of loadings investigated.

All of the ultimate stretch and work curves of Fig. 14 through 19 are concave upward--that is, the rate of loss was greatest during the first cycle and then became progressively less thereafter. It is pertinent to note that the concavity of these experimental curves is of the opposite sense to that contained in the theoretical work of Ragossnig (4,5). This author's theoretical concept of strength reduction may be expressed graphically by Fig. 21, wherein the rate of loss of strength becomes progressively greater with increase in number of applications. That is, the ultimate curve is concave downward. Though this concept has a plausible physical explanation (progressively increasing number of severed bonds in the sack paper under a repeatedly constant impact energy), it is a concept which apparently is not compatible with the laboratory repeated tensile test results obtained in this study.

It is also evident from Fig. 14 through 19 that the magnitude and rate of decrease of ultimate stretch and work are dependent on the level of the applied stress--the greater the input, the less the available stretch and

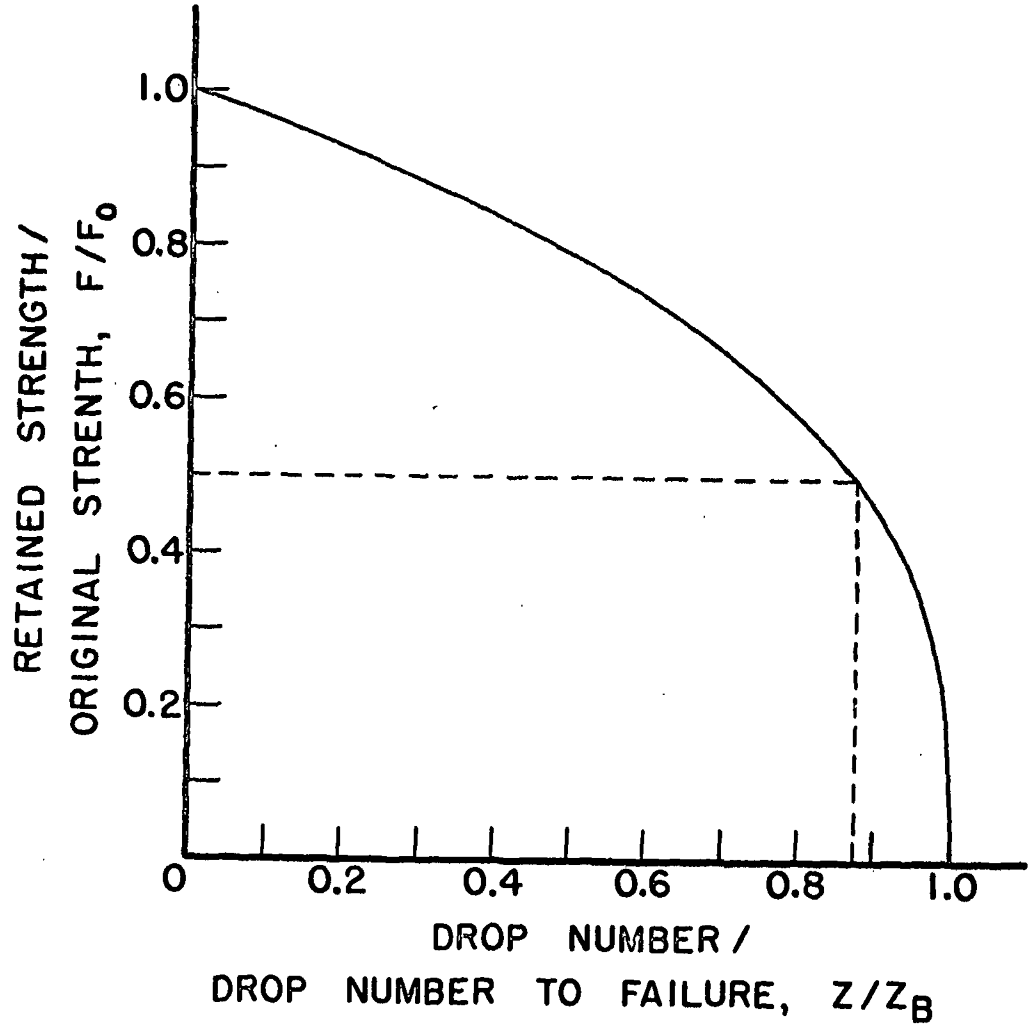

Figure 21. A Graphical Representation of Ragossnig's Hypothesis for the Decrease in Dynamic Strength of Sack Paper as a Result of Repeated Impacts

work and the more rapid is the loss--i.e., the shorter the fatigue life. For example, five applications of high-level load reduced the available cross-machine work to 52% of the value of the virgin paper (Table VII and Curve 17i) while five applications of the intermediate level reduced the available work to only 79%. This trend is evident throughout all of the processes investigated. This is reasonable in view of Fig. 20, for if the specimen is stressed to a high level $\underline{A'}$ (rather than $\underline{A}$) the stretch and work available at $\underline{B'}$ will be less than for the $\overline{OAB}$ cycle.

The preceding was concerned mainly with the changes in ultimate properties of sack paper under repeated application of constant load, elongation or work. The nature of repeated stress or strain is further illuminated by examination of the induced properties, that is, the load, elongation or work at the point $\underline{H}$ on the final ascent of the load-elongation curve of Fig. 12. These properties are described graphically by the dashed-line curves of Fig. 14 through 19.

Under the constant applied elongation and constant applied work processes (Fig. 15, 16, ~~17~~, 18, and 19), the induced load progressively increased with repeated application. Eventually, failure of the specimen occurred even though the tensile strength was sensibly constant. Thus, the demands on the material were progressively increased under constant elongation and work processes until finally the tensile strength was exceeded. This behavior is characteristic of visco-elastic material. Figure 22 is a tracing of cross-machine load-elongation curve of sack paper subjected to repeated constant increment of elongation, $\underline{e}_a$. The first cycle $\overline{OAB}$ results in the nonrecoverable stretch $\overline{OB}$. Since the non-recoverable stretch $\overline{OB}$ of Fig. 22 is lost after the first cycle, the second application of a constant elongation, $\underline{e}_a$, necessitates that the load must

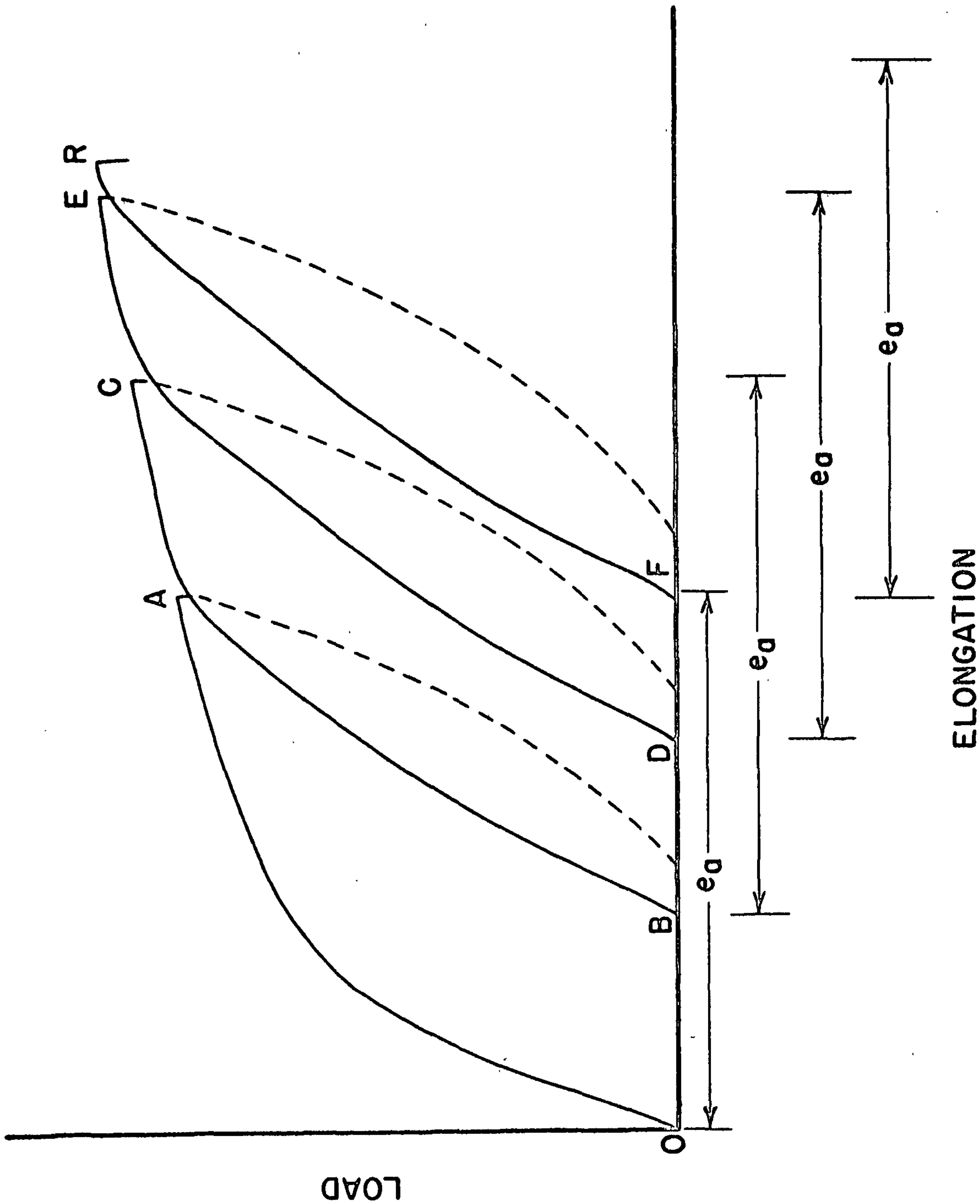

Figure 22. Illustration of Repeated Tensile Test with Constant Applied Elongation

increase in keeping with the load-deformation characteristics of the specimen until the applied elongation has been satisfied. Thus, the second application ($\overline{BAC}$) induces a higher applied tensile force than was associated with <u>A</u> on the first cycle. The second increment $\overline{BD}$ of nonrecoverable stretch demands that the applied tensile force at <u>E</u> associated with the third application of elongation, e_a, be even greater than the preceding.

Another interesting aspect of the applied properties is the near equivalence of the constant elongation and the constant work process. It may be noted that when the applied elongation was held constant, the associated applied work was also sensibly constant, and <u>vice versa</u>. For example, when in-machine elongation was repeatedly applied at a level of 63% of its initial value (Curve 15k), the induced work (Curve 15q) was 45 ± 1% of the virgin work. And when the applied work was maintained at 45% (Curve 16g), the in-machine induced elongation (Curve 16k) also was constant at 61% of the virgin stretch.

At first inspection this may appear to be an unexpected result. It might be anticipated that inasmuch as the elongation is constant for each cycle and the loads increase with each cycle that the induced work would also increase rather than remain constant from cycle to cycle. Examination of the slopes of successive reload curves, on the other hand, shows that the shape of the curves varies slightly from cycle to cycle. This is illustrated in Fig. 22, which is a tracing of a cross-machine repeated tension test where the applied elongation, e_a, was maintained constant. Evidently, the shape changes sufficiently that the progressively higher loads are compensated by a loss in work (area under the curve) due to the diminishing slope of the reload curves, $\overline{BA}$, $\overline{DC}$, and $\overline{FE}$. Computation of the area under the curve of each cycle of Fig. 22 gave the following values for induced work:

Cycle	Curve	Induced Work, in.-lb.
1	$\overline{OA}$	1.512
2	$\overline{BC}$	1.534
3	$\overline{DE}$	1.588
4	$\overline{FR}$	Rupture

The range of work values for the first three cycles is ± 3% of the mean value. A somewhat similar change of shape of the reload curves was noted by Ihrman and Andersson (20) in paper which had been repeatedly stressed by sack impact.

In passing, it may be of interest to note that the work associated with solely the reload phase of each cycle ($\overline{BA}$, $\overline{DC}$, and $\overline{FE}$ of Fig. 22) becomes progressively larger. For the specimen of Fig. 22 the work during each reload phase is

Cycle	Curve	Induced Work, in.-lb.	Increase Based on Cycle 2, %
2	$\overline{BA}$	0.676	--
3	$\overline{DC}$	0.806	19
4	$\overline{FE}$	0.892	32

As discussed in the Introduction, the work during each reload phase corresponds to the elastic component of work associated with the point of load reversal of the preceding cycle. These data demonstrate that each cycle to a higher load level introduces an additional increment of the total elastic work.

(It is characteristic of many viscoelastic materials that a reload curve, such as $\overline{BA}$ of Fig. 22, does not pass precisely through the point of load reversal, A, of the preceding cycle. The work values listed above terminate at the same total elongation as the maximum point of the preceding cycle. An alternative computation of work was made by terminating at the load level of the previous maximum point. This method resulted in greater values of work and greater per cent increases, namely, 26 and 48%, for the third and fourth cycles, respectively.)

In view of the strong experimental evidence of virtual equivalence between the constant elongation and the constant work, the three constant processes studied in this series of experiments reduce effectively to two types of processes: constant force vs. constant elongation (or work). As noted in the discussion of ultimate properties, there is a marked distinction in the behavior of the paper when subjected to these two general types of processes (Fig. 14 vs. Fig. 15 and 16; Fig. 17 vs. Fig. 18 and 19). Under the constant-load process, the sack paper apparently was capable of safely withstanding a large number of applications of load (in general, more than the fifteen applied in this study), even at the relatively severe 90% level of applied force. On the other hand, the constant elongation or constant work processes at the corresponding level frequently resulted in tensile failure after about five applications. Recognizing that the high level of applied force or applied elongation corresponds to the same point on the virgin load-elongation curve, it may be concluded that the constant elongation or constant work processes are more severe types of loading.

Comparing Fig. 14 and 17, 15 and 18, 16 and 19, it is seen that the deterioration of cross-machine properties is uniformly greater than the in-machine deterioration. For example, fifteen applications of high-level force (Fig. 17) reduced the cross-machine ultimate elongation and work to 49 and 44% of their respective virgin values, while the same relative severity of in-machine load (Fig. 14) resulted in reductions to only 77 and 71%, respectively. Similarly, five applications of the intermediate level of constant elongation (Fig. 18) reduced the cross-machine available stretch and work to 57 and 52%, while the in-machine stretch and work (Fig. 15) fell off to only 87 and 84% of their respective initial values. Further inspection of the curves reveals that the rate of deterioration was much greater with cross-machine than with in-machine specimens, particularly as a result of the first several applications of load.

It may be recalled that the cross-machine specimens were strained at twice the rate employed on the machine-direction specimen. According to previous studies (15,16), the faster the strain rate, the higher the elasticity and the lower the plasticity; thus, the effects of a higher rate of straining on the cross-machine tests would be in the direction of a lower fatigue or deterioration rate.

These trends are readily understandable in view of the relative shapes of the initial load-elongation curves for in- and cross-machine directions of sack paper, as illustrated in Fig. 23. The large initial stretch of sack paper in the cross direction is mainly attributable to the large plastic component of elongation. But the plastic component is rapidly dissipated during the first several applications of stress. That is, much of the virgin stretch of cross-machine paper is nonrecoverable, and thus one or two applications of load markedly reduces the available stretch and work.

It may be noted that the cross-machine specimens withstood no more than one application of the high levels of applied elongation or work. This simply means that such a large portion of the first application of elongation or work was nonrecoverable that the attempted second application soon exceeded the available stretch and work.

One must be cautious, of course, in projecting the results of this laboratory study to the performance of sacks. These tensile specimens were tested at a much slower rate than occurs in sack impact. The load-elongation curves of this sample of paper may be assumed to be quite different at high rates of strain and hence the deterioration effects may also differ from those experienced in this study.

Furthermore, the paper in a sack is subjected to biaxial (two-directional stresses). Relative to virgin uniaxial tensile properties, biaxial stresses may be expected to (a) increase the apparent stiffness, (b) decrease the stretch, and (c) change the tensile strength. Accordingly, the deterioration characteristics of sack paper under biaxial repeated tension may differ in magnitude from the uniaxial properties obtained in this study.

Moreover, these tests were performed with an arbitrary recovery period between successive applications of stress, which bears no simulation to either the laboratory sack impact test or, certainly, field service conditions. It is known, however, that paper exhibited a time-dependent recovery between load cycles which may be expected to influence its deterioration characteristics.

Lastly, these tests were performed on parent sack paper. The data do not speak, therefore, to the effects of creasing, sack fabrication, and sack filling on the behavior of the multiwall sack paper under repeated tensile stresses.

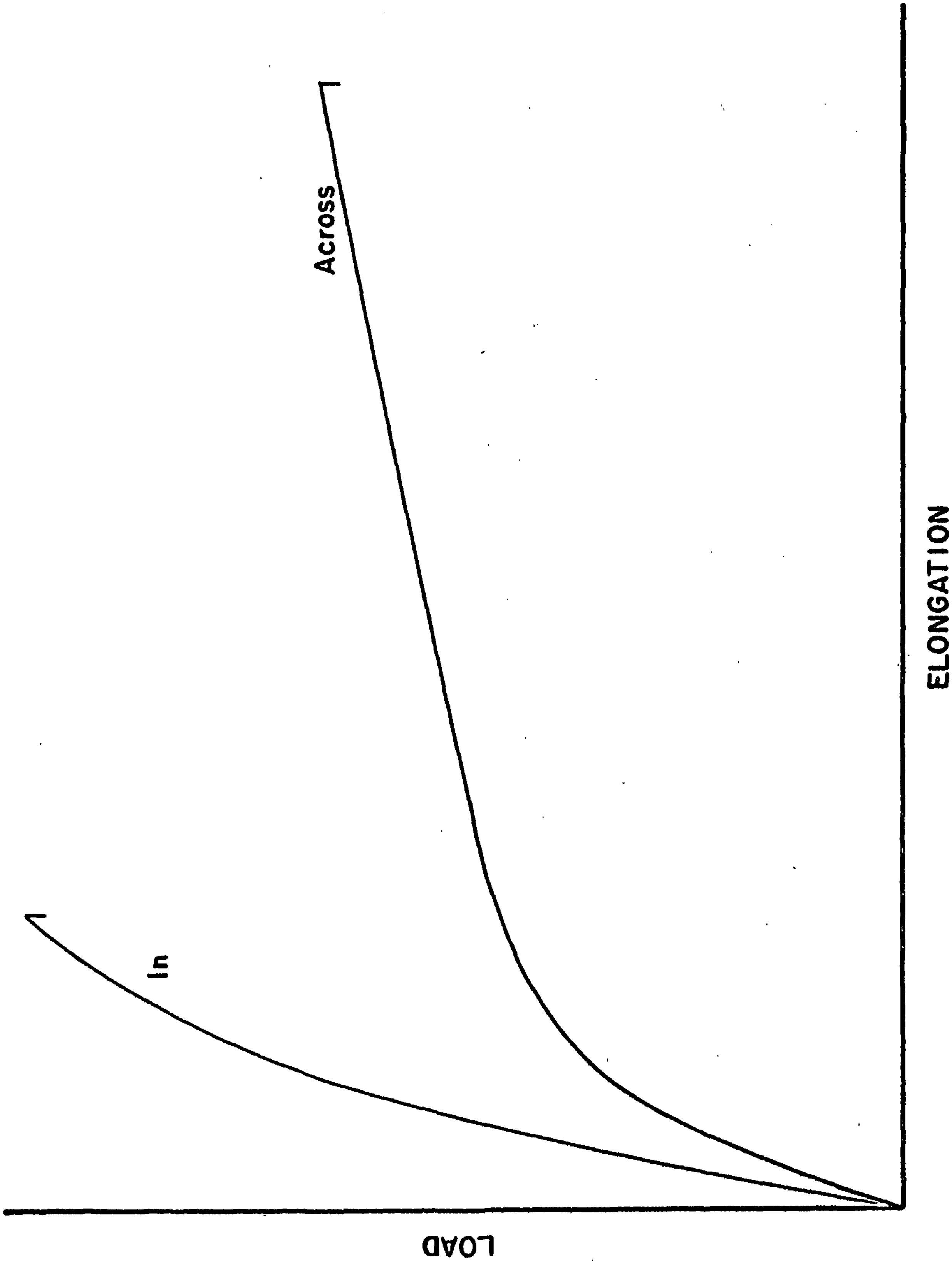

Figure 23. Representative Virgin Load-Elongation Curves for in- and Cross-Machine Directions

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